Why build energy models as MCPs? An economic perspective

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About KAPSARC

The King Abdullah Petroleum Studies and Research Center (KAPSARC) is an independent, non-profit research institution dedicated to researching energy economics, policy, technology, and the environment across all types of energy. KAPSARC’s mandate is to advance the understanding of energy challenges and opportunities facing the world today and tomorrow, through unbiased, independent, and high-caliber research for the benefit of society. KAPSARC is located in Riyadh, Saudi Arabia.

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Introduction

The discipline of microeconomics focuses on the outcomes of the actions of economic agents, where economic agents can be individuals as producers and consumers or organizations that deliver goods or services. Microeconomists have developed a collection of models to understand and represent these behaviors. Mixed Complementarity Problems (MCPs) are among the most recent such models.

This paper addresses the community of energy economists who are not necessarily focused on modeling but want to know more about current methods for representing economic equilibria. We describe in non-mathematical terms the advantages of using MCPs over other representations of markets and how to decide when they should be preferred to a more traditional (and possibly simpler) optimization approach such as linear programming.

Our focus is on partial equilibrium models - models of single or multiple economic sectors, but not of the whole economy. Examples of these models include the TIMES model housed in the International Energy Agency (Loulou and Labriet, 2008 and Loulou 2008), the US Energy Information Administration’s National Energy Modeling System (NEMS) (EIA, 2013), and the KAPSARC Energy Model (Matar et al. 2013, 2014), a multi-sector equilibrium model of the Saudi energy economy, “KEM-SA”. The experience of building a model that captures government interventions in energy markets within Saudi Arabia provides insights into the circumstances under which it pays to adopt an MCP formulation, rather than use optimization methods.

The most fundamental decision in building a microeconomic model is whether to model the outcomes of decisions, or the actual decisions the agents make. The standard answer is that agents’ actual decisions cannot be modeled when:

- there are many agents and they are heterogeneous in resource endowments and cost structures as producers of goods or services
- they place a multiplicity of values on products and services as consumers

In such situations, econometric methods can be used to model the outcomes of decisions. This involves gathering data on the outcomes and statistically estimating production functions and demand curves.

For instance, using econometric techniques for estimating end-use demand is usually the best strategy because consumers are heterogeneous and there are large numbers of them. They have a range of incomes, and value goods and services differently. The consumer response to gasoline price changes illustrates this. The outcome of gasoline consumption is a consequence of millions of decisions made by households, and each household has its special circumstances. Similarly, an estimate of a production function of the manufacturing sector is based on inputs of capital, labor, energy, and materials. An aggregation over all of manufacturing represents a multitude of processes and circumstances and the estimated production function captures any shift in the aggregate use of labor, say, as the costs of capital, materials, or energy increase.

By contrast, when a sector in a market has relatively few agents and the production and decision processes are understood, the market outcomes can be estimated by modeling the decisions, technologies, and cost structures of the agents. Some models use a mixed strategy, such as estimating demands for energy services econometrically and then modeling the choice of technologies to meet those services based on cost. Thus, the methodologies can be mixed together.
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When modeling decisions, one enumerates the choices that agents can make, develops data on the inputs, outputs, costs, and revenues associated with every choice. One then forecasts the outcomes that result from the agents making profit-maximizing or cost-minimizing choices. As an example, to build a model of the oil refining industry in a region, one would assemble data on the capital and operating costs, capacities, inputs, and outputs of all of the refinery units in the region and build a linear programming model of the region’s capacity, as if all of the refineries operated as one refinery. Models of this sort go by several names. They were originally called process models because production processes are explicitly modeled. Other names are bottom-up models because a sector is modeled starting at the “bottom” and is built “up”, and technico-economic models because of their explicit representation of technologies. We use process model because that is the original term chosen by Alan Manne, the inventor of this approach (Manne 1958).

This paper introduces the MCP approach with a simple example of a competitive market. It presents a framework to decide when to build an MCP model instead of using a standard optimization approach. We show what goes into building an optimization model of a competitive economy, explain the limitations of the optimization approach, and demonstrate how MCP models address the weaknesses of optimization models. We end with a description of how to build the MCP of an energy system, drawing on KAPSARC’s work in modeling Saudi Arabia’s energy economy.

Complementarity: A natural way to describe market equilibria

The classical law of supply and demand

Consider an idealized economy consisting of a single good for which demand and supply curves are given. Demand and supply are functions of prices, but economists usually depict them in inverted form: the demand curve shows the price at which consumers are willing to buy a given quantity of the good; the supply curve shows the price at which producers are willing to sell a given quantity of the same good.

The demand curve is downward sloping since consumers consume less when the price is higher. The demand curve is defined by the marginal value of consumption, i.e. the satisfaction (expressed in monetary terms) that the consumers get from consuming an additional unit of the good. For a given market price, consumers stop consuming when the marginal value of consumption equals the price. The supply curve is upward sloping since the producers produce more if the price is higher. Under the assumption of perfect competition, the supply curve is defined by the marginal cost: producers increase production until the cost of producing the last unit of a good equals the market price. The two curves intersect at a point called the equilibrium point, a price and quantity such that the quantities demanded and offered match at the given price. Figure 1 illustrates this classical “law of supply and demand”.

This classical representation, however, assumes that the supply curve is smooth and that the good is produced at the equilibrium. The reality might be more complex: at the equilibrium the good may not be produced, or the price may include an economic margin (or rent) if there are limits on production capacities. A full representation of the market then leads to the complementarity conditions, as illustrated in the next section.

MCP formulation of a market equilibrium

To explain what the complementarity conditions are, we modify the supply curve in Figure 1 to simplify the discussion. We replace the upward sloping
supply curve with a single supply step where all the producers use the same technology and have the same operating cost \( C \) and the total available production capacity is \( M \). We could add more steps to the supply curve to reflect the different costs of multiple producers. However, that would complicate the discussion without the benefit of any additional insight. In Figure 2 we show the three possible equilibria that can occur. In Figure 2a, the demand curve lies above the production cost for any quantity at or below capacity \( M \). All of the available supply is used and the market clears at a price equal to the marginal value of capacity, \( M \), for consumers. This price is greater than the operating cost, \( P_e > C \) which means that \( P_e \) includes an economic margin (also called “scarcity rent”) \( R = P_e - C \). In Figure 2b, at cost \( C \) there is more capacity than demand, the market price is \( C \), and there is no margin for the producers. Finally, in Figure 2c the cost of operating the capacity, \( C \), is always above the demand curve. Here there is no production and no margin.

As shown in Figure 2, either the unused capacity (Figure 2a) or the margin (Figure 2b) is 0. This is known as the complementarity condition. Table 1 summarizes the complementarity conditions representing these equilibrium outcomes, and therefore describes an MCP. The term “mixed” refers here to the fact that the quantity produced lies between a lower bound (0) and an upper bound (the available capacity \( M \)). Appendix 1 provides a formal representation of this MCP. If an algorithm finds the solution to the complementarity conditions in a model, then it has also found the economic equilibrium.

The limited capacity depicted in Figure 2 sometimes refers to a “resource”, in which case the margin made by the owner of that resource (beyond a market rate of return on investment) is commonly referred to as a “rent.” Rents permeate energy economies, including:

- coal mines with very thick seams have resource rents because the market price is determined by the production cost of thinner seams.

- existing electrical generation equipment has capacity rents when demand grows faster than anticipated because of time lags in building new capacity, or because favorable sites are no longer available.
Figure 2

Figure 2a: Production cost below demand curve
Source: KAPSARC

Figure 2b: Production cost intersects demand curve
Source: KAPSARC

Figure 2c: Production cost above demand curve
Source: KAPSARC
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– producers that are closer to their customers have location rents. For example, the mine-mouth prices of Appalachian coals, even after adjusting for heat content, are higher than the prices for coal produced in Wyoming because these coals are nearer to eastern customers.

Efficiency of the market equilibria

Assume that a benevolent planner decides the quantity sold in a market, taking into account both supply and demand information. Consider the situation described by Figure 1. By being benevolent, this planner chooses the quantity \( Q_e \) that maximizes the area between the supply and demand functions; it is the difference between the consumers’ accumulated willingness to pay and the producers’ cumulative cost and is called the economic surplus. Maximizing the economic surplus leads to choosing a quantity that is equal to the equilibrium quantity \( Q_e \). This area between the demand and the supply curves for \( Q \) between 0 and \( Q_e \) is given in Figure 1 by the triangle made by the supply and demand curves and the price axis. These concepts apply to Figure 2 with the different equilibrium values \( Q_e \) outlined in Table 1.

This shows that optimizing the economic surplus is equivalent to finding competitive market equilibria, and that an economy is efficient in the sense of providing the greatest economic surplus at the intersection of the supply and demand curves. This relationship between the surplus and the equilibrium is the formal expression of Adam Smith’s invisible hand, where the self-interests of the individual agents lead to the maximum social surplus. This maximum aggregate social welfare is realized by achieving competitive market equilibria.

Having illustrated why adopting a complementarity framework represents an intuitive way to describe economic equilibria, we now explain how using linear programming to find an efficient solution amounts to determining market equilibria. This is essential to understand the economic foundations and limitations of using optimization in building energy models.

Linear programming: A standard approach for equilibrium models

Linear programming was developed as a planning tool that has found its greatest use in optimizing the production of goods and services in large firms. The benefits come from centrally coordinating organization-wide production processes. From an historical perspective, Leonid Kantorovich (1958) invented linear programming in the Soviet Union to optimize the plans of the central planning agency and George Dantzig (1951, 2002) independently invented linear programming to optimize manpower planning in the US military.

<table>
<thead>
<tr>
<th>Case</th>
<th>Quantity produced</th>
<th>Margin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production cost below demand curve</td>
<td>( Q_e = M )</td>
<td>( R = P_e - C \geq 0 )</td>
</tr>
<tr>
<td>Production cost intersects demand curve</td>
<td>( 0 &lt; Q_e &lt; M )</td>
<td>( R = 0 )</td>
</tr>
<tr>
<td>Production cost above demand curve</td>
<td>( Q_e = 0 )</td>
<td>( R = 0 )</td>
</tr>
</tbody>
</table>

Table 1: The different equilibrium outcomes
Source: KAPSARC
Marginal costs, marginal values and rents in linear programming

Each activity of a linear program uses goods and/or labor or capital services to produce one or more outputs. If a production activity uses one or more inputs from sectors not included in the partial equilibrium model, those inputs have assumed prices that serve to determine the expenditures on those inputs per unit of production. Those expenditures become the cost coefficient of the activity. Each good and/or service used and produced by the activity is typically constrained and these constraints have dual variables, also known as shadow prices (i.e. the additional value that would be captured if the constraints were relaxed), in the terminology of linear programming. If the activity’s output(s) is sold to consumers (and not to other activities), the dual variable on the constraint that links the activity’s supply with demand equals the marginal value of the output for consumers. If the outputs are sold to other activities, then the same dual variable represents the marginal value of the intermediate good from the perspective of the purchasing activities.

The marginal value is also the economic gain that would result from adding an additional unit of output in the model at no cost.

At the optimal solution, the marginal value of an output is its implicit price in the model. Shadow prices have an interpretation of rent when they apply to a constraint that represents a limited resource. A rent associated with a limitation on the availability of a resource represents the value of an additional unit of the resource for the model, i.e. its implicit price. The rent accrues to the owner of that resource and it can be different from zero only if this constraint is binding. In addition, the activity can have direct costs appearing in the objective function. The marginal cost of operating an activity is the sum of the direct costs and the rents/prices paid for inputs. This defines a non-smooth supply curve like that appearing in Figure 2a.

For an activity that operates in the optimal solution, the cost of operating the activity (direct costs plus rents/prices paid) is equal to the marginal value(s) of the activity's output(s). In other words, marginal cost equals marginal value, which equals price. This is equivalent to the classical equilibrium described earlier for a perfectly competitive economy. If the marginal cost of running an activity exceeds the marginal value of the activity’s output, the activity does not operate, as shown in Figure 2c.

One of the first commercial uses of linear programming was a model of an oil refinery (Manne, 1958). Because the savings generated were so large, oil companies funded the improvements in linear programming software for the next couple of decades. During this early period, the power industry began using it for the planning of investments (Massé and Gibrat, 1957). Capacity expansion models became a standard application of linear programming in utilities.

The original uses of linear programming led planners to frame their thinking around centrally coordinating complicated organizations. Linear programming became an important tool for understanding markets after Paul Samuelson (1952) and Stephen Enke (1951) recognized that the algorithm Dantzig (1951) developed to solve linear programs, the simplex algorithm, was a method for finding an equilibrium in a competitive markets – a market where prices equal the cost of the marginal unit of production and where no firms exercise market power.

A linear program can be viewed in several different ways. The most common is to think of the model in standard algebra as a set of equations that express how the solution to the equations has to meet conditions that represent the coordination of activities in an organization. Another way to frame the model is in terms of agents/activities where each activity (i.e., each variable in the model) is treated as a Leontief production function that converts inputs into outputs in fixed ratios. This latter view is what we use here and was emphasized by Dantzig (1963).

We now show how the activity view leads to abstracting economic sectors as process models.
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Marginal value of capital and linear programing in the Soviet Union

To give a sense of the importance of prices in linear programs, in Kantorovich’s original model of the Soviet economy he included a constraint limiting the amount of capital to what was available at that time. This constraint had a dual variable that was positive. Since under strict Marxism capital had no value other than its embodied labor and its price should be independent of current constraints, Kantorovich and his work were hidden away and linear programming was avoided in the Soviet Union. Once it became clear that linear programming was an important planning tool, the Soviet bureaucracy rehabilitated Kantorovich to lay claim to its invention.

Every activity is an agent that can decide its production level, depending on whether or not it is profitable to produce. Supply functions are of the type described in Figure 2 with a constant marginal cost, possibly up to some maximal capacity $M$ and multiple steps in a step function with increasing costs on the steps. Substitution effects, that is, variations in the proportions of inputs and outputs are captured by having multiple activities with different proportions that can vary the mix of inputs and outputs in the solution. Demand functions in a linear program are either fixed quantities that do not vary with price, or unlimited demand at fixed price, or a step function with multiple steps that have lower prices with higher quantities. The functions in Figure 1 can be approximated arbitrarily closely by step functions, making linear programming a general tool for modeling more complicated supply and demand functions.

Constraints express limits on resources, demand requirements, product or input characteristics, or policy or technical restrictions. The levels of the activities in the optimal solution depend on the choice of the objective function. The objective functions most commonly used in linear-programming-based models of economies maximize economic surplus. If demand is fixed, minimizing cost produces the equilibrium, and if market prices are fixed, maximizing profits leads to the equilibrium.

Dantzig’s original method for solving linear programs, the simplex algorithm, uses a pricing mechanism to decide whether an activity should operate. The simplified version of the algorithm presented here gives a story for how a market reaches an equilibrium:

- Start with a set of trial prices with one price per constraint.

- For each activity, determine if it is profitable to produce by taking the trial prices of the constraints and the production coefficients.

- Increase the production of the most profitable activity until some other activity is driven out of business.

- Operating this activity at its maximum economic level changes the trial prices. The effect of the changes in the trial prices is that the marginal profit of the introduced activity is reduced to zero.

- At the optimal solution the activities not in the solution cannot enter profitably and, using the dual variables of the constraints as prices leads to zero profits for all positive activities, as would be the case in a competitive economy.

The properties of the linear-programming solution that we have outlined are the same properties of a competitive equilibrium. For every activity:
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- Supply equals demand at a set of market prices (this results from the surplus or cost objective function)
- Marginal cost equals marginal value, equals price
- No one can enter the market profitably with the available technologies (the profit from adding one unit of production is negative or zero for any activity not already producing)

The assumption that prices equal marginal costs/values is central to energy models based on linear programming like TIMES. An energy system is composed of multiple interacting firms (or sectors), each firm maximizing its profits or minimizing its costs. Assuming perfect competition, or that all the firms are controlled by a single planner, amounts to assuming that all prices of commodities traded among firms are marginal costs or marginal values.

When sectors are combined into a single model, as in TIMES, and one sector supplies another, these sectors are connected through material balance constraints where the supply delivered equals the amount consumed. The purchase cost in the consuming sector comes from the dual variable of the material balance constraint. This dual (the implicit price as explained above) is also known as the transfer price.

When a multi-sector model is built, often it is easier to build models of individual sectors and then combine them. Noting that the objective function of the model of a sector includes the cost of all inputs from outside the sector, the objective function of the combined model adds together the objective functions of the sectors that are combined together, after removing costs or prices that are determined internally within the multi-sector model but are external to the individual sectors. That is, a model of just the electricity sector includes fuel costs. Once fuel models are combined with the electricity model, the costs of fuels are dropped from the electricity model, and the fuel costs affect electricity-generation activities through the dual variables, or transfer prices, on the material balances that connect the fuel models with the electricity model. The resulting combined objective function can be optimized subject to the set of constraints faced by the sectors. In short, the key assumption implicitly made in standard process models is that transfer prices of commodities between firms or sectors are marginal costs/values. This allows the modelers to optimize a unique objective function representing the whole energy system.

To summarize, activities represent individual agents and the surplus objective leads to the market clearing. The different objectives of the firms or sectors can be combined into one single objective, because each activity in the model can be considered a price-taking agent that takes a positive value only if it is profitable – given the shadow prices on the constraints – independent of which firm has that activity in its production process. The solution is a competitive equilibrium because marginal cost/value equals price.

Limitations of optimization in representing economic sectors

Sometimes optimization methods cannot be used to represent markets.

To find the equilibrium by maximizing the economic surplus requires computing the difference between the area under the demand curve(s) and the area under the supply curve(s). These areas are integrals in calculus terms. Although the areas exist when the model represents a single product, or the demand curves have the property that no change in a product price affects the demand for another product, the areas may not exist in commonly occurring situations where demand and supply need to be represented by a system of equations that does
not satisfy these properties. When the area below demand curves does not exist, demand is said to be “non integrable.”

A measure of the response of the demand for a product to prices is known as the price elasticity, the percentage change in the product’s demand as a function of the percentage change in the product’s price and the prices of competing products. In models of markets with multiple products the value of the elasticity of demand for product 1, given a change in the price of product 2, is normally different from the value of the elasticity of demand for product 2 given a change in the price of product 1. As a property of integral calculus, these elasticities have to be identical for the integral to exist and demand to be integrable. Price elasticities can also be introduced for supply curves involving several products, with similar properties needed to guarantee that supply is integrable.

The strength of optimization in representing competitive markets is also its weakness. In many markets, the price does not equal the cost of marginal units of supply. Although optimization can also represent markets with monopolists, other markets without perfect competition need to be modeled. Some examples include:

- When a few firms dominate a sector, they can exercise market power and set marginal revenue, rather than price, equal to marginal cost
- In this situation each of the large firms has its own profit-maximizing objective function, and if the firms are not identical, the objective functions cannot be added together into a single appropriate objective function in an optimization model (unlike a monopoly model where the objective function of the model changes from maximizing surplus to maximizing profits for the monopolist)
- An industry can operate under particular pricing rules, as with price indexation clauses that link gas and oil product prices in long-term contracts or price escalation clauses that link the cost of a project to price indices
- Governments can impose regulations that alter prices. They can impose caps on the prices of some essential commodities, impose taxes, or grant subsidies.
- Examples also include administering the transfer prices of fuels among industrial sectors in Saudi Arabia, and charging consumers the average cost of electricity under traditional rate-of-return regulation.
- Linear or nonlinear programs that maximize economic surplus cannot represent certain kinds of taxes.
- They can represent taxes that are a fixed amount (excise taxes). However, governments usually impose taxes that are a percentage of the price, as with a value-added tax. Linear or nonlinear programs cannot represent that tax because the tax level needs to be known in advance when determining the coefficients in the objective function.

Before the development of good algorithms to solve MCPs, a collection of algorithms was developed that involved solving linear programs, adjusting the solutions, and resolving the linear programs until the market equilibrium was found. Murphy, Sherali, and Soyster (1982) showed how to use this approach to find the equilibrium in markets where oligopolistic producers can exercise market power. Hogan (1975) addressed the problem of asymmetric cross-price elasticities, and Greenberg and Murphy (1985) presented algorithms that deal with the tax problem and government price controls. All of these algorithms require computational skills and modeling efforts that are not required when MCP tools are used. For example, to deal with asymmetric cross-price elasticities, starting from a trial solution:
An approximation of the demand curve is constructed that runs through the trial solution and cross-price elasticities are set to 0.

After a trial equilibrium is found, the new trial prices are used in the demand curve with the cross-elasticities, providing new trial quantities.

A new approximate demand curve is constructed through this new price-quantity pair and the cycle is repeated until successive trial solutions are within a tolerance.

In a specific example of trying to capture the effects of average-cost prices for electricity, the difference between the marginal and average cost of electricity in a trial solution is subtracted from the transportation cost of the transportation activity that links the generation regions to the demand regions. This difference is updated at the same time the demand approximation is updated. These adjustments fall into the category of Gauss-Seidel algorithms.

An important advantage of MCPs is that the model is completely separate from the solution algorithm. With optimization-based approaches the model and algorithm have to be mixed together, making model management harder.

**Modeling using the MCP framework**

In formulating an optimization program, one writes the variables and equations that represent the physical flows and the costs of each step in making and delivering products. The economic rents and prices are calculated as part of determining the levels of the activities undertaken by the agents, as we have discussed. In linear programming, the linear program with supply, production and demand activities and constraints on resources, demands, etc. is known as the primal problem. MCPs represent the decisions of individual agents on whether to produce or not in the same way as linear programs. In addition MCPs can contain supply and demand functions that use functional forms that cannot be included in optimization models. The MCP, therefore, provides a more general description of a competitive market than the linear or non-linear models that maximize economic surplus. Furthermore, MCPs enable a higher degree of generality because they can also accommodate representations other than purely competitive markets. This is done by including an explicit statement of price relationships and complementarity conditions different from the price relationships in these markets as outlined in Table 1, as the rents and prices appear directly as variables.

As an example of price relationships, when firms exercise market power the pricing formulas do not set marginal cost equal to price. They set marginal cost to marginal revenue, taking into account that a product price is lower at a higher level of demand. In MCPs, when a sector is subject to rate-of-return regulation, the pricing formula calculates the average cost of electricity. With value-added taxes, the delivered prices are the supply prices multiplied by one plus the tax rate.

Every optimization program can be written as an MCP. For instance, the pricing formulas in a linear program can be written explicitly as the dual linear program, a program where the dual variables are the variables and where every constraint is associated with an activity in the original, primal, linear program. The MCP consists of the constraints from both the primal and dual linear programs and the complementarity conditions, which require that a dual variable can be positive only if the associated constraint is binding. However, not every MCP can be written as an optimization program. Appendix 2 provides a very simple example of an MCP that cannot be represented as an optimization problem.

Optimization is still preferred when the market structure and government policies meet the right conditions because building a linear programming model takes less time to build than an MCP. This is
because the prices are found in the solution to the primal linear program and formulating the dual linear program and the price relationships is therefore unnecessary. An additional advantage is that the solution algorithms for linear programs are faster than MCP solvers and can handle larger models.

Building MCP models

KAPSARC has developed a formal procedure for building MCPs. This is valuable because multi-sector equilibrium models are large, complicated models that have to be put together from smaller pieces. Since MCPs require that the pricing relationships be written in addition to the production activities, we first build linear programs, convert them into MCPs and then introduce the features that are different from competitive markets. The steps for building a model or sector from scratch are:

- Estimate rough initial values for prices and quantities to facilitate model construction
- For each sector, build a primal linear program separate from the full model, debugging the competitive model
- Using the same coefficients as in the primal program, write the dual linear program
- Run both primal and dual programs, checking that the values of their objective functions are equal (so called “strong duality property” of linear programs) and that the solution values for prices and quantities in the primal and dual match (beyond degeneracy issues)
- Take the constraints from the primal and dual models and write the complementarity conditions to form the initial MCP
- Modify the pricing rules and add regulatory conditions
- Build transportation linkages to connect the sectors and run the full model

KAPSARC has a project underway to develop software that automatically generates the dual equations in the modeling language that we use, GAMS. This will eliminate what turns out to be a very time-consuming step. An experimental program that is available through the GAMS Development Corporation (GAMS, 2014) produces an MCP from an optimization program after the full model is generated from the GAMS equations. This approach is not completely satisfactory in that the mnemonics that tell the modeler where an activity or constraint fits in the general model structure are lost, and the model cannot be altered to include regulatory conditions.

We are developing a model management system that includes a database of model components in the form of GAMS equations that are separate from the data. This means, for example, we can take the general model statement for the electricity sub-model for Saudi Arabia (part of KEM-SA, described below) and reuse it for other countries, modifying the model structure as necessary, adding the new model structure to our collections of models, and producing new data for the other countries. As part of this model management system, we are developing databases that contain the input data to the model and an archiving system to allow the retrieval of models and runs used in our studies.

KEM-SA as an example of MCPs

The assumption of prices equal to marginal costs or marginal values does not hold in the case of Saudi Arabia. Transfer prices of fuels among energy (or energy-intensive) sectors are administered to lessen the losses for electric and water utilities and to favor the development of certain industrial activities, such
as petrochemical production. These sectors are dominated by large organizations, suggesting price regulation as an appropriate approach to control market power.

The extent of the price controls leads to the power and water (desalination) utilities buying crude oil from Saudi Aramco at a price lower than US$5 per barrel, whereas the marginal value of crude oil consumed domestically is the export price (US$107.8 per barrel on average in 2011 for Arab Light Crude). The actual marginal value of a barrel of oil may be lower than the export price, since Saudi Arabia is a major oil exporting country with significant spare production capacity, but it is certainly above US$5 per barrel. Table 2 gives the prices of fuels for power and water production and feedstocks for the local petrochemical industry in 2011. In addition, the administered price may depend on the consuming sector. For instance, cement companies buy heavy crude at US$6 per barrel and power utilities buy heavy crude at US$2.67 per barrel.

The current Saudi energy system cannot be represented as a straightforward linear program with a unique objective function, but it can be represented as a regulated competitive equilibrium. The fuels demanded by the consuming sectors are supplied at the administered prices by the upstream oil and gas sector or the refining sector.

KAPSARC developed an MCP formulation that allows us to simulate an economic equilibrium over the Saudi energy system with inter-sector administered prices. KEM-SA is solved as a single MCP, with a workflow designed to facilitate decentralized component development as described in the previous section. KEM-SA is the first use of an MCP formulation for a portion of the Saudi Arabian economy.

Our formulation allows us to analyze policy options for the redesign of the pricing structure in the energy sector in a way that the Saudi population would perceive as fair, while inducing conservation and optimal technology choices. It provides a framework to test potential policies that lead to more efficient

<table>
<thead>
<tr>
<th>Product</th>
<th>Price</th>
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<tbody>
<tr>
<td>Natural gas</td>
<td></td>
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<tr>
<td>Methane</td>
<td>0.75 USD/mmBtu</td>
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<tr>
<td>Ethane</td>
<td>0.75 USD/mmBtu</td>
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<tr>
<td>Crude oil</td>
<td></td>
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<tr>
<td>Arab Light</td>
<td>4.24 USD/barrel</td>
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<tr>
<td>Arab Heavy</td>
<td>2.67 USD/barrel</td>
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<td>Petroleum products</td>
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<tr>
<td>Diesel</td>
<td>3.60 USD/barrel</td>
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<tr>
<td>HFO 360cst</td>
<td>2.08 USD/barrel</td>
</tr>
<tr>
<td>Propane*</td>
<td>650 USD/metric ton</td>
</tr>
<tr>
<td>Naphtha</td>
<td>830 USD/metric ton</td>
</tr>
</tbody>
</table>

Table 2 – Local prices for the domestic power, water, and petrochemicals sectors in 2011

*Prevailing CFR naphtha prices in Japan minus 28%
Source: National Commercial Bank (2012), Council of Ministers Resolution No. 55
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**Figure 3:** The sectors represented in the KAPSARC energy model and the major flows among the sectors
Note: end-use demand is defined here as all energy consumption not captured in the sub-models
Source: KAPSARC

**Figure 4:** Layers of modeling capabilities
Source: KAPSARC
equilibria, while retaining either current or alternative administered prices. A detailed description of the model and its calibration is provided in Matar et al. (2013). Below, we summarize the key features.

KEM-SA is a partial-equilibrium model that integrates into one model six sectors, each one captured in a sub-model. The full model is obtained by concatenating all primal and dual linear programs of the sectors that were initially described separately, and then altering the dual to represent prices that are not marginal costs, following the steps outlined above. The model can be run as a single-period, long-term static equilibrium model, or as a multi-period model with a user-specified planning horizon. The equilibrium is found using GAMS and the PATH solver.

Figure 3 shows the sectors and the flows among them. Every sub-model can also be run separately, taking exogenous prices for inputs used in the sector and exogenous quantities for outputs demanded by other sectors. The power and water sectors, which meet only local demand in Saudi Arabia, are modeled as cost-minimizers. The refining, cement and petrochemicals sectors – which also export and import – are modeled as profit-maximizers facing fixed local demand and capped exports. Oil and gas production is taken exogenously from Saudi Aramco’s production data. The oil and gas upstream sector is, therefore, represented with available supplies at administered prices and a pipeline structure that minimizes the cost of meeting regional demands for gas and crude oil.

Using a long-term static version of the model calibrated to 2011 data, we analyzed various scenarios in comparison to a current-policy baseline. One of these scenarios, termed the “Price-deregulation scenario”, measured the impacts of pricing fuels at their marginal values. The marginal values of crude oil and oil products consumed domestically were set at the 2011 export prices by the model. This particular scenario sets the upper limit for economic gain from changing transfer prices and could have been formulated as a standard linear program.

As shown in Matar et al. (2014), the current fuel consumption could be reduced substantially, without altering consumer prices while preventing the annualized net cash flows of the utility sectors from falling below zero. The policies examined could have potentially generated economic benefits exceeding US$23 billion in 2011, or about 4% of Saudi Arabia’s GDP. This economic benefit comes mainly from inter-sectoral fuel-pricing policies that incentivize shifting the mix in technologies used in generating electricity and producing water to more efficient choices.

Conclusions

This non-technical overview of the modeling choices and their economic implications is intended to broaden the understanding of MCPs in the energy community. It aims to assist modelers in making modeling choices and considering whether to build MCP instead of optimization models. We have shown some of the uses of MCPs. One of the ways of using MCPs we have not explored here is in optimizing policy parameters that alter market outcomes, or in representing certain games such as Stackelberg games where there is one leader and multiple followers that make choices knowing the leader’s actions. These models are known as mathematical programs subject to equilibrium constraints. Here the parameters or the leader’s decisions are optimized subject to the equilibrium solution of the MCP. Figure 4 illustrates the nested layers of modeling capabilities. More thorough discussions of MCP modeling and solution methods can be found in the books by Nagurney (1992) and Gabriel et al. (2012).
Appendix 1: Formal expression of the complementarity problem

Let $P = mv(Q)$ be the (inverted) demand function, where $mv$ stands for marginal value. Finding the economic equilibrium described in Figure 2 is equivalent to finding $Q$ and $R$ that solve the following statement which involves one of the three complementarity conditions:

- $mv(Q) - C - R = 0$ and $Q = M$ and $R > 0$ (Figure 2a)
- $mv(Q) - C = 0$ and $0 \leq Q \leq M$ and $R = 0$ (Figure 2b)
- $mv(Q) - C < 0$ and $Q = 0$ and $R = 0$ (Figure 2c)

Using the $\perp$ symbol as a mathematical shorthand for expressing complementarity, the above statement is equivalent to finding the $Q$ and $R$ that solve the following system of inequalities:

(i) $0 \leq M - Q \perp R \geq 0$
(ii) $0 \leq C + R - mv(Q) \perp Q \geq 0$

Where:

(i) is equivalent to: $M - Q \geq 0$ and $R \geq 0$ and $R (M - Q) = 0$. This states that there is an economic rent if – and only if – the limit on production is reached.

(ii) is equivalent to: $C + R - mv(Q) \geq 0$ and $Q \geq 0$ and $Q (C + R - mv(Q)) = 0$. This states that the good is not produced if the marginal cost exceeds the marginal value, and that the marginal value is equal to the marginal cost plus an economic rent if the good is produced.

Appendix 2: A simple MCP with government intervention

In the example of the economy with a single good, we assume that, as a social goal, the government does not want the price of the good to exceed the value $P_{up}$. To make sure that production always meets consumer demand, the government pays a subsidy per unit of the good sold if the marginal cost of meeting demand at the capped price $P_{up}$ exceeds $P_{up}$. This subsidy, $S$, is defined as the amount of the marginal cost of production that is subsidized.

Let $P = mc(Q)$ and $P = mv(Q)$ be the inverse supply and demand functions. $mc$ stands for marginal cost and $mv$ stands for marginal value. The price the consumer sees after the subsidy is $P - S$. The possible market equilibria resulting from the government intervention can be described by the following MCP:

(i) $0 \leq P_{up} - mv(Q) \perp S \geq 0$
(ii) $0 \leq mc(Q) - S - mv(Q) \perp Q \geq 0$

Where:

(i) states that there is a subsidy if – and only if – the cap on price is reached.

(ii) states that the good is not produced if the marginal cost less the subsidy exceeds the marginal value, and that the marginal value is equal to the marginal cost less the subsidy if the good is produced.
There are three possible equilibria, described by Figures A1, A2 and A3 (where equilibrium quantity and subsidy are $Q_e$ and $S_e$ respectively):

**Figure A1:** Price is subsidized by the amount $S_e > 0$
Source: KAPSARC analysis

**Figure A2:** Market clears below the price $P^{up}$
Source: KAPSARC analysis

**Figure A3:** Demand is 0
Source: KAPSARC analysis

The equilibrium conditions of this MCP do not correspond to the optimality conditions of any optimization model.
Appendix 3: Building a linear complementarity problem

The formal statement of a linear problem using matrices and vectors is as follows:

\[
\begin{align*}
\max & \ v \cdot Q \\
A \cdot Q & \leq b \\
Q & \geq 0
\end{align*}
\]

The corresponding dual linear program:

\[
\begin{align*}
\min & \ R \cdot b \\
R \cdot A & \geq v \\
R & \geq 0
\end{align*}
\]

Where \( R \) is the vector of dual variables (rents) associated with the constraints in the primal program. Let \( Q^* \) and \( R^* \) be the solutions to the primal and dual programs. We have:

\[
v \cdot Q^* = R^* \cdot b
\]

\( Q^* \) and \( R^* \) are the values of \( Q \) and \( R \) that satisfy the following MCP:

\[
\begin{align*}
0 & \leq b - A \cdot Q \perp R \geq 0 \\
0 & \leq R \cdot A - v \perp Q \geq 0
\end{align*}
\]

This MCP is similar to that given in Appendix 1. It states that a dual variable can be positive only if:

- the associated constraint is binding
- a non-binding constraint has a dual variable equal to zero, an activity can be operated only if its marginal cost equals its marginal value
- a non-operated activity has a marginal cost greater or equal to its marginal value.

These complementarity relationships are also known as the Karush-Kuhn-Tucker optimality conditions of the primal linear program.
References


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