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An introduction to the KAPSARC Toolkit for Behavioral Analysis (KTAB) using one-dimensional spatial models

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About KAPSARC

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In this paper we will introduce the reader to the concept of the quantitative analysis of Collective Decision Making Processes (CDMPs), sometimes called negotiation or bargaining. Our objective is to provide insight into the discipline for those new to the subject and to provide a toolkit for those already familiar with CDMP models.

Our approach will examine the theory, data requirements, practical modeling "blocks," and post-solution analysis. The paper is arranged in seven sections as described below. Each section starts with a short summary followed by a more detailed technical discussion of the topic.

The software libraries for the KAPSARC Toolkit for Behavioral Analysis (KTAB) are available as an open source download at http://ktab.kapsarc.org

1. Introduction to KTAB.....................................page 5

The section discusses what KTAB and CDMPs are, who the platform is being designed for, how it can be used, and the objective behind building KTAB.

KTAB is freely available, open source, state-of-the-art software and software libraries that have been designed to enable the rigorous and systematic analysis of CDMPs.

2. Introducing one-dimensional spatial models.....................................page 7

The section explains what is meant by a CDMP and introduces the example used throughout this paper. It sets out a simple way of visualizing the different positions advocated by the actors involved in a CDMP and provides a base level of understanding of the merits and requirements of our approach to enable engagement with KTAB.

Simple and transparent analytical models often outperform expert judgment. The well-tested "spatial model" of collective decision making assesses the positions of actors on a linear spectrum; the balance of influence is then assessed to estimate the final outcome.

3. Feeding the models: Input data.....................................page 12

This section outlines the data requirements for this and other types of CDMP and outlines a few ways the data can be collected. It also helps to embed the ability to fashion model-appropriate questions and to collect the relevant data for a much broader range of cases.

The key data for the one-dimensional spatial model are the range of plausible policy choices, the identities of major stakeholder groups, their positions on the issue, and the amount of influence they are likely to exert.

4. The theory behind one-dimensional spatial models..page 18

Important concepts are explained that will be used in subsequent sections and which have wider applicability across decision science. CDMPs are grouped into three high level categories and the logic behind two important theories (the Median Voter Theorem and the Central Position Theorem) is introduced. This section aims to cement the basic elements of theory upon which the models are founded and to provide a theoretical basis for the conclusions drawn from a KTAB analysis.

The analytical models of collective decision making focus on identifying the policy option which the group would take over any other. Different sub-models of how much influence actors will exert are called “voting models”. Depending on which precise voting model is used, some simple formulae can be derived to estimate the outcome.

5. Exploring one-dimensional spatial models..page 24

This section provides the foundations for building incrementally more complicated and nuanced models based on models pre-built in the initial release of KTAB. The various approaches are explained both descriptively and mathematically to ensure that, while descriptions may be simplified, the mathematical underpinning remains rigorous.

Two different voting models are explored, via four examples. The effects of alternative parameterizations are compared. One comparison contrasts giving powerful actors a weight greater than or equal to weak actors. Another alternative compares the effect of actors’ willingness to compromise and accept risks (or not); the representation of this difference in terms of utility models is explained. Step-by-step instructions for the calculations of each example are provided.

6. The KTAB framework..page 42

This section provides greater depth for the specialist who may wish to go further and build custom models.

The formal notation for describing voting models and domain-specific utility models is presented in order to derive the forecasting rules of section 5. The notation is illustrated by describing not only formal elections but also generalized exertion of influence.

7. Post-solution analysis...page 48

This section describes some forms of sensitivity analysis, successful uses of the spatial model to analyze real-world problems, and some limitations common to all formal analyses. The importance of using sensitivity analysis to address these limitations is emphasized.

Beyond the problem of running the analysis is the question of interpretation. This section allows the applied user and the recipient of the model results to understand what those results might actually mean.

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1. Introduction

1.1 Section summary

KTAB is freely available, state-of-the-art software that has been designed to enable the rigorous and systematic analysis of Collective Decision Making Processes (CDMPs).

KTAB is being designed with three types of users in mind. The program code is available directly for computer programmers. Power users will be able to construct their own specific models based on the existing structure, and applied users will be able to access pre-built models through a simple graphical user interface.

Collective decision making processes are those in which a group of individual actors interact to arrive at a single decision. Common examples include the deliberations of corporate boards. These processes have usually been studied in a purely qualitative fashion, but there is a growing body of evidence which suggests that computer models can deliver additional insights. KTAB is being designed to unlock these additional insights for a broader range of analysts, but its release is also hoped to extend the awareness of computer models as a route to the investigation of CDMPs, and to prompt a wider acceptance and uptake of a quantitative approach to such analyses.

1.2 KTAB introduction

KTAB is an open source platform for building models to allow the systematic and rigorous analysis of CDMPs. KTAB has been developed by KAPSARC in order to meet the need for widely available, state-of-the-art, supported, and open-source software that facilitates the modeling and analysis of collective decision making.

KTAB is currently developed as libraries of code to enable analysts and computer programmers to construct their own models of CDMPs, secure in the knowledge that their approach is theoretically sound. These are already available for free download. A domain specific language (for power users who understand the theoretical underpinnings) and graphical user interface (for the applied user) are both currently under development. In order to broaden KTAB's audience and to encourage a more widespread adoption of using computer models to improve understanding of CDMPs, KTAB also comes with prebuilt models that represent both established techniques and novel applications.

Together, these allow the non-technical user to produce reasonable analyses of the potential outcomes of CDMPs involving a range of modeled groups. Example groups might be the top management in a firm, senior power brokers in a public entity, and so on. In any group, the actors hold different values, positions, and views, which they bring to the CDMP.

A CDMP may be defined as any situation in which a group of actors engage in order to arrive at a single outcome. In both general and technical parlance, CDMPs may be termed bargaining or negotiating. To side step legacy interpretations of these words we adopt the term CDMP, but the general meaning remains the same. The deliberations of a corporate board, the internal debates of ruling parties, the voting of electorates, and the haggling between seller and buyer at a market stall can all be viewed as CDMPs, albeit with different actors operating according to different rules.

CDMPs have traditionally been viewed as a subject for qualitative, not quantitative or model based, study. Most commercial negotiators, politicians, and courting couples instinctively reach for their intuition, experience, and knowledge, not scientific journals or statistical primers. Yet the power of models and their ability to overcome bias while providing additional insights to intuition, experience, and knowledge, have been demonstrated repeatedly in a range of equally qualitative fields.

Models have long been shown to be better diagnosticians than clinicians (Goldberg 1965), are better at predicting which projects will run into trouble than the procurement managers paid to make such judgments (Snijders et al. 2003), and can even give insight into the stability of personal relationships (Dawes 1979). In some situations "experts perform worse with growing experience" (Tazelaar and Snijders 2004). Models really can be an extremely useful adjunct to expert judgment.

We believe that KTAB can deliver similar benefits to the understanding and analysis of CDMPs. By channeling experts' qualitative knowledge along a systematic and quasi-quantitative route, and by using models to analyze that knowledge in a structured manner in accordance with transparent principles, we argue that KTAB can bring great insight to research into the probable or plausible outcomes of CDMPs. KTAB allows analysts to choose from a series of models, or build custom ones, that identify which from among the available options in each group is a plausible outcome for the CDMP. Analyses of this sort can be, have been, and are being used to support deliberation by those within modeled groups, those affected by modeled groups, and interested third-parties. The scope of application and usefulness of these kinds of models is very broad.

There is also a much broader objective. Decision science and the modeling of negotiations remains a niche subject, with little traction outside its academic practitioners. Yet people engage in attempts to forecast the outcomes of group interactions in myriad different roles. KAPSARC's hope and intention is that KTAB will be material in helping to realize the very considerable potential inherent in these models for expanding their range and scope of application, and introducing them to a broader audience.

Our aim is to explain the use, application, and construction of models with the aid of KTAB by introducing a particular example of a CDMP taken from the published literature. The example presented is intentionally simple: it is a one-dimensional spatial model of a real world example with accessible data. It is important to remember that there is no magic bullet for analyzing plausible outcomes of collective decision making. What we are presenting here is a systematized way of approaching these problems in order to arrive at a better, more robust understanding.

2. Introducing one-dimensional spatial models

2.1 Section summary

Simple and transparent analytical models often outperform expert judgment. The well-tested "spatial model" of collective decision making assesses the positions of actors on a linear spectrum; the balance of influence is then assessed to estimate the final outcome.

The value of KTAB is that we can construct formal models to analyze CDMPs.

The first step is defining the question and who the actors are, but beyond this the models we will present in this discussion of CDMPs all rely upon three basic inputs for each actor. The actors' attitudes towards the question at the heart of the CDMP are referred to as the actors' Position. Their power to get their own way in the CDMP is described by two other variables. Influence describes the power they can bring to bear if fully motivated; Salience describes their level of motivation. These two parameters are combined to give the actors' actual Exercised Power.

Together these three input parameters (Position, Influence and Salience) allow us to build a series of models of surprising value. It is fair to ask why we should build any model at all. Collective decision making processes are parts of human behavior; most people may not intuitively understand the need for, or benefit of, such models. Experience and expertise are what most rely on to assess how human interactions will work out. Evidence is mounting to suggest that this faith is misplaced. In a variety of situations, from family counselling to clinical diagnoses and project management, models have proven themselves as extremely insightful adjuncts to human intuition. The reasons behind this are comparatively simple. Humans can pursue rigorous, logical trains of thought, but more often they rely on simple rules of thumb that can be swayed by hunches giving disproportionate weight to unimportant factors. An additional complication arises from a dislike of testing our own assumptions for fear of being wrong. A computation model has no such limitations.

Collective decisions are distinct from other types of decisions in that they arise through coordinated interaction. Market prices arise from groups, but the individuals within these groups do not coordinate their actions, nor do they try to influence one another – they simply act according to their best interests given the situation they find themselves in. In collective decision making processes, the actors seek to gain an advantage by persuading others to build coalitions.

As different actors take different positions, so these disparate views can be portrayed graphically as points on a line. Left wing politicians could be arrayed at the left and right wing politicians on the right. The important point is that positions are arrayed in a relative manner so that actors that hold similar views with regard to the issue at stake are grouped graphically. A very famous paper from political science was written by Bueno de Mesquita in 1984. In it he presented a series of studies using this graphical approach. For the sake of simplicity we will adopt one of his examples as the case study for this document, introducing the concepts of our models and the capabilities of KTAB through an exploration of his data.

2.2 Basic variables

To begin with, it is helpful to define three input properties, and one derived quantity, of each actor in one-dimensional spatial CDMP. These are termed Position, Salience, Influence and Exercised Power (this last quantity being derived from the previous two).

Position

A number of actors collectively decide which of a set of options, ordered on a single spectrum or scale, will be chosen. In modeling one-dimensional spatial CDMPs, each actor begins the CDMP advocating a particular outcome. That advocated outcome (which may or may not be sincerely held) is called the Position of the actor and it is assigned a numerical value according to its place in the ordered spectrum of all options (the range of advocated outcomes).

Each actor has a Position, and some actors may share the same Position. The possible outcomes of the CDMP are, or at least include, the Position of the actors.

Influence

An actor's Influence is a measure of the degree to which the actor is able to affect the outcome. It is a measure of power relative to the other actors if fully motivated.

Salience

People place different weights on specific issues. An issue that is "a matter of life and death" to one actor may be "no big deal" to another. Salience is an indicator of how important the issue is to the actor.

Exercised Power

A derived quantity from an amalgam of the actor's Salience and Influence. Influence measures the potential power of an actor to affect the results of the CDMP, and Salience measures the actor's willingness to exercise that power on this issue. Together, they combine to give us an index of an actor's actual sway, its Exercised Power, in influencing the CDMP.

2.3 Why do we need a model?

Models can take many forms. In this document we consider a model to be a simplified representation of relevant factors and their relationships. Models are often, but not always, mathematical.

Examples of where models outperform expert judgment abound. It has long been documented in clinical environments (Goldberg 1965). Simple models can tell us much about the stability of marriages. Even bad models (in which the various factors are given random weightings) have been shown to pick "better" students than admission tutors (Dawes 1979). Procurement managers are bested by computer models when it comes to identifying which projects are likely to run smoothly and which will run into trouble (Snijders et al. 2003).

Experience and expert judgment do not hold all the answers; in fact they can be a drawback : in some situations "experts perform worse with growing experience" (Tazelaar and Snijders 2004). In part this may be because of the ways people process data and make decisions. Human decision making has been split into two complementary systems: intuition and deliberative thinking (Kahneman 2005). Intuition is easy, relying on heuristics honed by experience; deliberative thinking requires much more effort. It turns out that everyone, experts and novices alike, benefits from doing a bit more deliberation and a bit less intuitive snap decision making (Moxley et al. 2012). Experts, with all their experience and knowledge, too quickly slip into using their intuition to make decisions. They consider less information and more quickly sort cues by relevance than novices do (Shanteau 1992); the experience of age brings on a similar tendency (Mata et al. 2012). Experts rely on simpler strategies and do less deliberative thinking (Pachur and Marinello 2013). The result can be suboptimal decision making.

So it is better not to rely on expert judgment alone when looking into CDMPs. Forcing ourselves to engage in systematic and deliberative thought can boost the rigor of our research endeavors. Unfortunately, deliberative reasoning can be undermined by cognitive wiring that "may not be consistently efficient at detecting conflicts during reasoning" (Pennycook et al. 2012). Polite words for a potentially devastating problem.

Many social problems (including CDMPs) rely on integrating individuating information with background data. For instance, we may know some statistics about a certain population but also some specific facts about a particular individual. When we start making decisions about that individual, we are likely to overweight the personal details, and underweight the statistics. We can find ourselves led by our prejudices, particularly when we can fit “objective data” to them, more than by statistics. Models don't make this error. They may not always pick up on the telltale cues an expert can, they may not know all the individuating information, but models give statistics their due regard.

Another thing that models do not do is overestimate themselves. Humans have a tendency to place too much faith in the accuracy of their own judgments, and seldom naturally seek information that would show themselves to be wrong (Nickerson 1998). This is known as confirmation bias and is one of the most well-documented foibles of human reasoning. Our dislike of cognitive dissonance, of being wrong, means that we have a tendency to cherry-pick information that shows us to be right. It makes us feel better, but does not do anything for our predictive powers.

Analyzing CDMPs requires researchers to coordinate an array of qualitative individuating information, often without any apparent timely feedback mechanism to enable the researcher to benchmark their accuracy. In other words, CDMP analysis contains all the cognitive pitfalls outlined above. In such circumstances, models, even simple models with random weightings, can often derive better results. Humans employ rules of thumb with error; computers just employ the rule of thumb.

An added dimension to the benefit of using models is that it isn’t necessary to build complicated computer models to take account of every piece of information. Not only can simple models outperform expert judgment, simple models can also outperform complex models (Haldane 2012). For this very reason, the models presented in this document are deliberately simple.

2.4 What is a Collective Decision Making Process?

Not all collections of individual decisions are instances of collective decision making. The market sets a price for commodities that is the product of countless decisions by countless different actors. Yet these decisions are primarily taken in response to individual preferences with little attempt to coordinate with the other actors involved. In fact, in many countries, it is illegal to take such coordinated actions. The models KTAB supports are not designed to address the outcomes of such uncoordinated mass decision making. Instead, these models are specifically designed to give insight concerning the outcomes of coordinated collective decision making, or at least decisions taken by collectives which take account of the preferences of their members.

Collective decision making is not straightforward. Alone, an actor can make the optimal choice according to their own preferences. However, in a group context, actors usually have to persuade others or are persuaded by them. It is not enough simply to represent individual preferences in such a model; the relative Influence of different actors must be taken into account. Weak or isolated individuals may be overruled and have to tolerate a collective decision with which they disagree. To prevail, coalitions will likely be formed and concessions granted. Both require realistic assessments of the preferences, strengths and weaknesses of the various actors. It is for this that the modeling capabilities in KTAB have been designed.

Political scientists who have worked on these sorts of models have often focused on parliamentary and legislative systems or the stakeholders in interstate conflicts. KTAB is designed for a much broader range of situations and actors. The actors involved in collective decision making do not need to be politicians, and certainly do not need to be democratically elected legislators. They could be members of a boardroom committee; they could be school friends; they could be nothing more than a social or economic group with a shared interest. They could be debating the launch of a new product or the division of a pile of sweets. We can view CDMPs as involving actors exercising Influence in an attempt to gain a desirable outcome, and CDMPs themselves as strategic interactions in which the decisions of the actors affect the payoffs for all involved (Morgan 1984).

2.5 Visualizing actor preferences

One way to portray the preferences of the various actors is to show them graphically in a technique referred to as spatial preferences (Black 1948). Actors' preferred alternatives are represented as points on a line depicting a range of choices. Other points on the line hold varying levels of Attractiveness for each actor. This means that even though actors hold particular preferences, they are often also prepared to compromise within certain bounds. This can reveal zones of possible agreement.

We will illustrate these concepts with an analysis from Bueno de Mesquita (1984a) of policy concerning the degree of government control over a national economy. His example provides a particularly accessible and well-known instance of using a formal model of a CDMP to forecast negotiated policy outcomes. We have purposely taken our example from the literature and not from a *de novo* KTAB example because we want to show that the types of models KTAB uses are well established and have been used to good effect in the past.

As KTAB is primarily a tool for constructing models (rather than being itself a model), showing that one-dimensional spatial models are analytically useful will reassure as to KTAB's usefulness as a tool for creating such models. It is not our intention to recreate Bueno de Mesquita's underlying models, nor to critique or verify his analysis. We are simply using his example, with its clearly defined issue and readily available data, as an aid to our exploration of one-dimensional spatial models and the capabilities of KTAB.

Published during the 1980-1988 Iran-Iraq War, and seeking to model decision making on the Iranian side, the paper identifies 27 actors of interest. Bueno de Mesquita (1984a) uses the term "group" for what we call individual actors; we will usually reserve the phrase "whole group" to denote the collection of all actors in the current situation. In his paper, each of the actors is given a two or three letter abbreviation for its name.

We shall use these abbreviations in what follows to identify the actors in the models. For the purpose of our explanation the exact identity of these actors is not important, though it is important to note that they represent a mix of formal and informal groups. Some groups are labeled by the name of their leader, while others (such as rural peasants, religious scholars, or various ethnic groups) carry a more generic label.

In the years before the Iranian Revolution, the country's economic and industrial development had been rapid, but by 1978 growth had slowed. While the internal debate was phrased in terms of prioritizing growth versus social justice, the new Islamic government nationalized both industry and the financial sector. The economy moved to a system of central planning with government controlled prices. The eight-year Iran-Iraq war, during which the paper was written, cost the country hundreds of thousands of lives and perhaps half a trillion dollars. 1984 was also the year in which the US imposed its sanctions, further isolating and weakening the economy. It was with this background of tensions that Bueno de Mesquita posed his investigation.

Bueno de Mesquita (1984a) provided two forecasts of economic policy. We will focus on the second, which gives results based on the Positions for 15 actors simply because the data set is smaller and easier to visualize. For now, we will present the actors' Positions in Figure 2.1 on a single issue.

Although it becomes progressively harder to visualize, this system can be extended to multiple dimensions as more elements in the negotiations are modeled. This extension from one-dimensional questions to multidimensional ones is not straightforward but these theoretical complications are beyond what is necessary for this paper.

Figure 2.1 shows the actors' Positions arrayed in response to a clearly defined question. In this example the question is: "What is the attitude of each group toward the role of the government in the economy?" It is critical that one frame the issue narrowly enough that experts can assign the actors' Positions along a single spectrum without saying "it depends…" Balancing against this is the need to keep the questions broad enough to remain of interest. Indeed, framing the question is a critical part of the analysis.

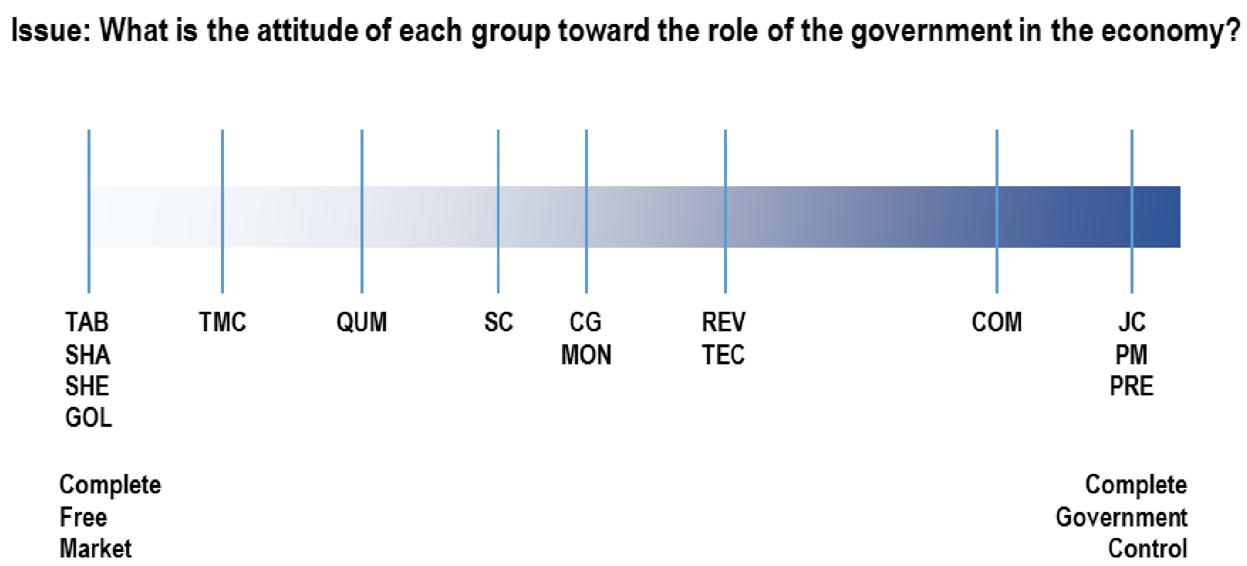

Figure 2.1: We reproduce Bueno de Mesquita's (1984a) representation of the actors involved in his investigation of economic policymaking in Iran following the 1978 revolution. He poses a simple question: "What is the attitude of each group toward the role of the government in the economy?". The attitudes of the various actors are then arrayed along a line. This spatial arrangement is what gives spatial models their name. Although we use the word 'actor', actors need not be individuals – they can also be stakeholder groups or institutions. In this instance, actors are identified with two or three letters (e.g. MON for Ayatollah Montezari, TMC for Tehran Militant Clerics, QUM for Qum Clerics, SC for Supreme Court, LCR for Lower Class Rural Peasants); see Bueno de Mesquita (1984a, Table 1) for the full listing.
Source: Bueno de Mesquita (1984a)

3. Feeding the models: Input data

3.1 Section summary

As stated, the key data for the one-dimensional spatial model are the range of plausible policy choices, the identities of major stakeholder groups, their positions on the issue, and the amount of influence they are likely to exert.

No model has much value without good data. A difficulty for models of CDMPs is that the data cannot be looked up in government or industry year books; websites do not publish tables of values ready for use in KTAB-style models. Instead we must create the necessary data by converting disparate sources of qualitative information into a single, systematic quantitative dataset. The easiest way to do this is by interviewing subject matter experts. Similar knowledge could also be gained through extensive surveys of relevant literature, such as newspaper reports or social media feeds. Regardless of the source, what is critical is that we begin the analysis with a clearly defined question and list of actors. Actors in this case are stakeholders : they need not be individuals but can be institutions, companies, or even informal social groupings.

In order to properly define the CDMP's underlying question, we must construct a spectrum of Positions. This is a line along which are arrayed various descriptions of competing Positions. In the example we have taken from Bueno de Mesquita, we would therefore array different views of how much control over an economy should be given to the state versus the market. Free marketeers could occupy one extreme; state planners the other. Intermediate descriptions would occupy intermediate Positions along the spectrum. This approach allows us to assign a numerical value to each described Position (and all undescribed Positions) relative to their Position along the line.

As already noted, actors need not be individuals but can also be formal and informal groups, so long as each defined actor behaves as a unit with one voice. With a question, a spectrum of Positions and a list of actors defined, the next step is to assign values to their Position, Influence and Salience. The measures are all relative, so it is easiest to assign values on a 0 to 100 scale. However, this is not necessary and Bueno de Mesquita uses a different scale. While Position and Salience are relative scores, Influence is relative and additive : two actors with Influence of 30 could combine to block an actor with Influence of 60. This can make Influence the hardest value to properly calibrate.

3.2 Sourcing the data

As explained in section two, the basic input data for the one-dimensional spatial models supported by KTAB and explored in this paper are the actors involved, and their Position, Salience and Influence. What has not been discussed is where the data comes from or how it is gathered. How do we identify the actors and assign the Positions, Influence and Salience scores?

In some situations the list of actors is obvious. Board members are listed in company accounts; government officials are usually highly public figures. In other cases the list of actors may be less clear. The stakeholders in many CDMPs include lobbyists who prefer to keep the degree of their involvement discrete.

It is important here to clarify that actors need not be individuals and certainly not highly-placed individual people. While such people are obviously important in many CDMPs, there are also actors which are really stakeholder groups, like the "businessmen", "rural peasants", "urban professionals", or "religious scholars" in the Bueno de Mesquita (1984a) example we are presenting and which is set out in Figure 2.1. In other situations "scientists", "the middle classes", "people concerned by air pollution", "financial traders", "northeast coal miners", and so on could all be listed as actors, in addition to named individuals. What is important is that individual actors act in unison on the modeled question.

Even where we can identify the actors, in the vast majority of cases it is not possible to interview all the different actors directly. Instead we must rely on experts who have studied the actors or closely followed the background to the CDMP. Through structured interviews we can identify the spectrum of possible outcomes, list the actors involved as stakeholders, and assign each a Position, Influence and Salience score.

A commonly heard discomfort is that it is not possible to know the mind of third party agents. How could anyone know what the Supreme Court members (actor SC in Bueno de Mesquita [1984a]) were really thinking? There are two possible answers to this question.

The first is that we seek to identify and score the actor's revealed preferences. We may not know their mind, but we can follow their acts and words. Even the most secretive of actors will leave a few clues by way of reported past actions or words. It is these clues that an expert will have collected and studied in the course of their careers, which must be sorted and analyzed to derive the various scores. Such sorting and analysis is not, of course, performed by a computer or model. This is the qualitative and subjective value that the expert brings to the process: we turn their qualitative understanding into quantitative data.

This in turn leads to the second way of looking at the problem. To a certain extent we are not seeking some incontrovertible or objective truth. This is probably eternally unknowable. Instead we are seeking an internally consistent view of the situation as it relates to the question to be analyzed. In a narrow sense, we are then answering the question of what the expert's view of the CDMP implies the most likely outcome is. We can extend this by aggregating the datasets of many experts, benchmarking and normalizing the various sets of scores to provide a single dataset we can then use as the basis of our analysis.

While structured interviews with an expert are among the simplest ways to gather data, they are not the only route. The analysis of large corpora of text (for instance, newspaper archives or historical social media feeds) is another route to identifying the actors involved and their revealed preferences.

3.3 Defining the spectrum of Positions

This is a two-step process. First we define what we call the Theoretical Spectrum of Possible Positions (TSPP). This does not imply any special doctrinal theory, rather the broadest range of Positions that can be imagined regardless of whether or not any actor actually holds them. Second we identify the range within which we expect the actors to cluster. This we refer to as the Practical Spectrum of Plausible Positions (PSPP). Most analyses are conducted using the Practical Spectrum of Plausible Positions. Nonetheless, by first defining the TSPP we have the opportunity to place individual CDMPs in a wider context.

3.3.1 The theoretical spectrum of possible Positions

Once the question is properly defined as a one-dimensional problem it is necessary to identify the two extreme possible Positions. For example, Bueno de Mesquita (1984a) addresses the issue of government control of the economy, so we consider what the theoretical limits on either side are:

- **Free markets:** The most extreme version of "free markets" might be thought of as an anarchist's paradise, because it would allow freedom not only in ordinary markets such as food, housing, and so on but also in several kinds of goods and services not usually allowed. For example, most societies do not allow unregulated free markets in addictive non-pharmaceutical drugs; buying, selling, producing, or even possessing them is illegal and subject to heavy penalties. The extreme of free markets could also eliminate legally enforced exclusion of private actors from provision of policing, courts of justice, or heavy weapons. Modern nation states provide such public goods almost exclusively through the state and very strictly control the scope of private actors.

- **Government control:** The most extreme version of "government control" of the economy would be similar to some of the most extreme experiments in control undertaken in the last Century. During the Russian Civil War (1918-1921) the policy of War Communism saw the Bolshevik party ban almost all private buying and selling of goods, even consumer purchases of food – citizens were directly issued rations by the state. Essentially all markets and trading were made illegal and subject to heavy penalties.

The practical range of plausible Positions will lie between these extremes. See Figure 3.1.

3.3.2 The Practical Spectrum of Plausible Positions

Once the TSPP has been defined, identifying the PSPP is reasonably straightforward. The expert simply identifies a range that will contain all the protagonists. In doing this there is no requirement for an actor to actually occupy either extreme of this narrowed range; it is simply to identify the range of the more plausible.

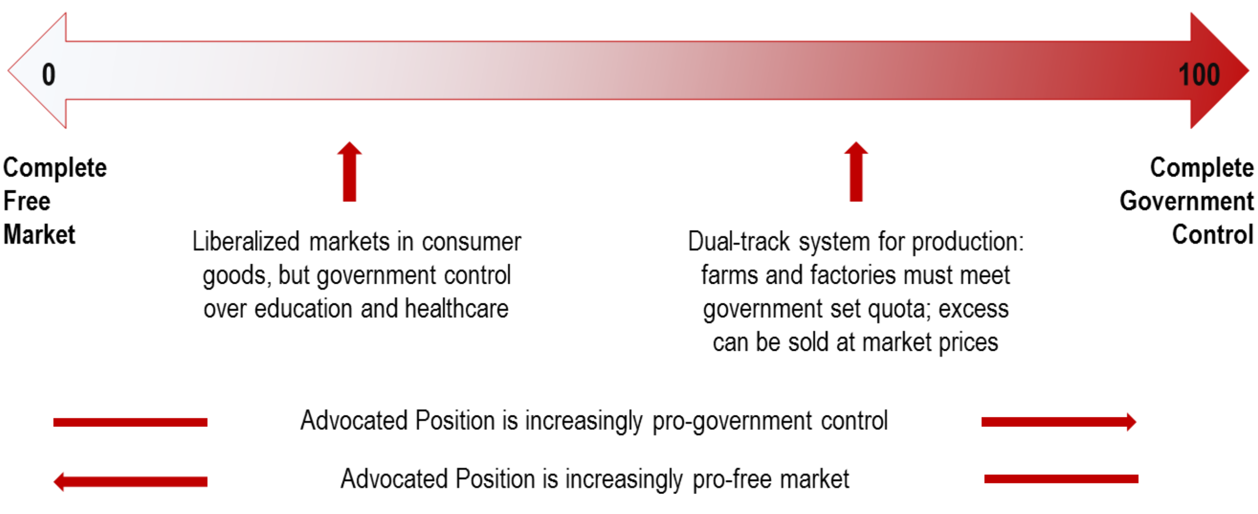

Figure 3.1: Constructing a Spectrum of Positions, using the economic policy case-study introduced in section 2.5 as an example. Qualitative descriptions of possible Positions are quantified on a numeric spectrum. The spectrum displays attitudes with regard to a single question or issue with movement in either direction signifying incrementally more extreme views. The end-point descriptions are taken from the original paper (Bueno de Mesquita, 1984a); intermediate Positions are example possibilities and bear no relation to the actual analysis or the Positions advocated by the listed actors in real-life.
Source: KAPSARC

Identifying the practical spectrum as a subset of the theoretical spectrum allows us to do three things. First, we can very quickly identify where the conversation actually is. Dissension aside, are the actors generally belligerent, are they generally more self-effacing, or is the battle raging over the center ground? Second, by working just with the practical spectrum, and normalizing the values of the identified intermediate Positions so that they run from 0 to 100 on a linear scale, we give ourselves more space to distinguish between the Positions of various actors. Third, if we are analyzing a problem that is one we have looked at previously and will investigate again in the future, then any shifts of the PSPP against the static TSPP will help us identify if previously unthinkable options have now come under discussion, and vice versa.

In our economic example, the PSPP is contained in the TSPP, but Bueno de Mesquita (1984a) unfortunately does not describe the limits of the PSPP precisely and provides no examples of intermediate Positions on the PSPP. Therefore, we provide an illustrative discussion of how Positions on the economic PSPP might be developed.

As mentioned, most societies ban a similar set of undesirable markets and insist on government domination of a similar set of sensitive markets. These can be taken as the limits of the PSPP: western regulated markets and 1980's style Soviet socialism. Between these two, there is a wide range of Positions along the spectrum of the plausible. For example, one extreme Position on food production is that all produce belongs to the state and will be taken from farmers without compensation. Another Position is the quota system, in which producers can produce whatever they want for the private market, but only after they have delivered their quota to the state. Another Position is that farmers produce only for the private markets, with no quota, but they must pay a fixed percentage income tax to the government out of the proceeds. The most free is where farmers simply produce whatever they want for the private market, with no quotas, taxes, or regulations.

A similar range of Positions exists on the consumption side. One extreme is that citizens are simply given free rations by the state, with no buying or selling involved. An intermediate Position is that common consumer items, such as bread and gasoline, are heavily subsidized by the state to ensure affordability. Another is that only a few critical goods and services, such as education or health care, are subsidized. The other extreme is that free market prices and quantities prevail in legal goods and services.

Having identified 0 and 100 on the Position line, the next step is to describe intermediate Positions. One way of doing this is to describe what Position 50 would look like, and then 25 and 75. In each case we are seeking a few phrases that can be used to help us identify whether or not we are at that particular Position. In this economic policy example, there are strong economic interactions between production and consumption that make it impossible to pick and choose every aspect of policy independently. Thus, each overall level of government control of the economy could be described by a few phrases outlining one coherent package of policies, and these could then be arrayed on the spectrum of plausible Positions. An alternative approach is to start with a set of intermediate descriptions and then work out their Positional score. The more detail the spectrum can be given (the more intermediate Positions described) the better. These will act as the signposts to aid the expert to assign the actors their Position scores.

The spectrum needs to be viewed as a scale where distance measures the change in consequences for the actors: the gap between Positions corresponds to the difference in outcome. An implicit assumption is that all actors roughly agree on the consequences of

Positions. The consequences of moving from Position 25 to Position 50 would be roughly the same magnitude as moving from 50 to 75. In other words, the spectrum requires reasonable calibration.

We are focusing on one-dimensional models and therefore require only a single spectrum. The Position represents a sliding scale of a single factor: a cartel's decision on the sales price to offer, a nation's bellicosity in international disputes, and so on. In multidimensional negotiations, a separate practical spectrum must be defined for each dimension.

3.4 Who are the actors involved in the CDMP?

The actors are all the stakeholders who contribute to the negotiations in some way. The list of actors does not need to be restricted to the few decision makers that step into a negotiation chamber. It includes all the stakeholders that will try to lobby for a particular outcome, whether they be formally organized groups or not. The more exhaustive the list, the better the Position and Exercised Power landscape will be mapped, but there is of course a trade-off between an exhaustive list and the expert's knowledge.

As set out in section 3.2, actors can be individuals or aggregates of individuals. Aggregates can be formal (such as a corporation) or informal (such as loose affiliations based on interests, for example young men sharing a love of fast cars). Each actor is a unitary entity speaking with one voice. Where an aggregate is home to conflicting voices, each with a different Position and the ability to exert Influence in different directions, the aggregate cannot be regarded as a single actor. Instead it is better broken up into its constituent voice – each one becoming an actor.

Where two individuals speak with the same voice, they may be brought together and treated as a single actor. Imagine a mediaeval ruler who espoused only the opinions of a trusted counselor, an actor no one but the ruler directly interacted with. There is no need here to model the counselor as a separate actor: the voice is heard through the ruler; the ruler has no voice but the counselor's. They become an aggregate, treated as a single actor.

3.5 Scoring the actors

3.5.1 Position scores

Once the PSPP has been defined, the expert needs to assign each actor a Position on the spectrum in accordance with their understanding of the actor's revealed preferences.

The range of scores typically runs from 0 to 100, as defined by the PSPP. For computational convenience, this is usually normalized to a zero-one scale when doing calculations as in section 5.

The Position scored is the actor's initial Position: the one they carry with them into the negotiation chamber. The expert needs to avoid trying to second guess the outcome of the negotiations and score what they think the actor will settle for. While various actors may shift their Positions as the negotiations proceed, the expert need only identify their initial Position.

This is most easily understood as the Position that people identify the actor as advocating regardless of whether or not the actor has explicitly said that it is their preferred Position. The Position may not be the actor's true desire. It is what they are prepared to stand by as revealed by their actions.

3.5.2 Influence scores

How easily can the actor Influence the CDMP if fully motivated? This is not a measure of how likely the expert thinks the actor's preferred Position is to win, nor is it a measure of the actor's motivation to win. Influence assumes that the actor will bring his full resources to winning the negotiation and takes account of all his formal and informal powers.

The simplest approach is to set the range of scores to run from 0 to 100. All actors are scored relative to the most influential (scored 100). Actor scores are also additive. While using a 0 to 100 range is the simplest approach, other scales are equally acceptable as long as the properties of relativity and additivity are respected. In the example introduced in section 2.5, the Influence scale runs from 0 to 12.4.

The combination of relativity and additivity can make Influence the most cumbersome score to derive. Each actor's score needs to be calibrated against all the other actors.

An example will help explain. The leader of an organization is likely to be the most influential and is scored 100. He has two immediate subordinates. If either could block the leader alone, they would also be scored 100. If neither could block the leader alone but can easily do so if they combine forces, they could be given the score of 70 each. 100 is greater than 70, but less than 70+70=14: the leader triumphs over each subordinate individual but not when they combine.

If the two subordinates combined can only just block the leader then they are scored 50 each (assuming they are equally influential): 50+50=100. If even when the two subordinates join forces they cannot block the leader then their score will be less than 50: e.g. 45+45=90.

Where do these numbers come from? Once again, they come from a subjective view of their Position in the CDMP's power hierarchy. Calibration can then be attempted by comparing the Influence of individuals against hypothetical coalitions.

This relative and additive approach to the score applies all the way through the list of actors involved. This can result in very low Influence scores being given to the weaker actors. The expert needs to resist the temptation to give senior subordinates high scores commensurate with some view of their formal seniority. Similarly, just because a collective actor has many technically excellent resources at hand does not necessarily mean that the actor has great Influence – the resources might be inapplicable to the conflict at hand, or they could be misapplied because of strategic misconceptions. The score is intended always to represent the actor's true ability to Influence the outcome relative to all others.

3.5.3 Salience scores

How much does the actor care about the issue in general? How motivated are they to exert Influence to produce their preferred outcome if and when the issue arises? One way to begin answering these questions starts with the observation that each actor has a portfolio of issues to which they pay attention. Salience identifies the importance of this issue in that portfolio, recognizing that people have an implicit budget constraint on exerting their Influence across the portfolio.

The Salience score is not the amount of time that an actor will devote to the negotiations, but rather their willingness to use whatever Influence they have to convince others of the merits of their own preferred Position. It is not their Influence; merely their motivation when the issue arises.

The range of Salience scores runs from 0 to 100:

- 0 to 10: the actor hardly cares and may not be aware of it
- 10 to 20: the issue is minor, but the actor is aware of it
- 20 to 40: the issue is one of many issues
- 40 to 60: the issue is among the top 3 or 4
- 60 to 80: the issue is the most important but there are still others that need attention
- 80 to 100: the issue is that actor's top priority

4. The theory behind one-dimensional spatial models

4.1 Section summary

The analytical models of collective decision making focus on identifying the policy option which the group would take over any other. Different sub-models of how much of an actor's exercised power will be exerted between different options are called 'voting models'. Depending on which precise voting model is used, some simple formulae can be derived to estimate the outcome.

At a very high level, there are three basic types of CDMPs:

- static
- simple dynamic
- dynamic with variable utility

This paper will focus on the second type. Static CDMPs can be seen in legislative elections; dynamic CDMPs with variable utility can be seen as extensions of the simple dynamic type, though the underlying algebra is fundamentally different and much more complicated.

Any model of CDMPs will have a forecasting rule. This is derived from our view of how negotiations actually occur in practice and determines how the winning position is selected. Different actors will lend alternative possible outcomes varying levels of support depending upon how attractive they are. Obviously, the actor's preferred Position is the most attractive. Different forecasting rules and different ways of modelling attractiveness give rise to many different models, but all can be called Generalized Voting Models. Voting is nothing more than the exertion of influence to promote or resist possible outcomes.

One of the most celebrated results in political science is the Median Voter Theorem (the word voter here does not imply any notion of Western style voting). The actors in the CDMP are presented with a series of pairwise comparisons. At each stage, actors support the Position closest to their preferred Position. For instance, 49% of the population could be far right, 49% far left. The far right would prefer a centrist government to a far left one. The far left would prefer a centrist government to a far right one. A compromise Position will emerge that has only 2% directly advocating it but can still gain overall support compared to either extreme. In very simple terms this is the median weighted Position, which is then declared the "winner" of the CDMP.

The Median Voter Theorem is clearly a stylized simplification. Most CDMPs see Influence exerted in a much more nuanced fashion. An alternative to the Median Voter Theorem that allows actors to support different Positions to different degrees is found in the Central Position Theorem. While before actors could vote just "yea" or "nay", the Central Position Theorem allows proportional voting, in which actors can lend one Position 30% of their support, and another just 10%, based on the attractiveness of each Position to each actor. As each position gains a certain percentage support from each actor, one Position will emerge with the highest overall support. The forecasting rule would then declare this Position, with the highest weighted attractiveness score, the winner.

4.2 General models of negotiation

At a very high level, CDMPs may be grouped into three categories.

- The first kind is a static, one shot choice between given alternatives, as is the case in many elections. Clearly, the final outcome of the CDMP is not necessarily one to which all actors agree.

- The second kind is a simple dynamic CDMP in which actors can generate a series of proposals and counter-proposals until an outcome is reached. Actors craft proposals that improve the likely outcome from their perspective, while winning enough support that the whole group prefers the new proposal to the old. Under certain technical conditions, one option can emerge which dominates all other outcomes in pairwise comparisons; we call this the Condorcet Winner (Condorcet, 1785). Once the Condorcet Winner is proposed, no other proposal can displace it: actors know that further proposals are pointless, and the CDMP finishes with the Condorcet Winner as the final outcome. The Condorcet Winner can be expected to persist until some event occurs to upset the balance of Influence. Again, the final outcome of the CDMP is not necessarily one to which all actors agree: weak actors might be overruled by strong ones. While the final outcome might be "the best of both worlds" that pleases a large number of the actors involved, it might equally well be an unhappy compromise that was far from ideal for most actors.
- The third kind is a more complex dynamic CDMP, but this time actors can induce other actors to change their fundamental perception of utility, and hence their Position. This changes the likely outcome of the CDMP (Pen, 1952). The complex bargaining process is fundamentally different and much more complicated than either of the first two kinds listed above. In this process, stronger actors can build coalitions in order to induce (coerce or entice) weaker actors into actually changing their preferences and advocated Positions. These attempts at inducement may or may not succeed: the weaker actors may simply concede, counter with an offer to make limited concessions, or even make their own attempt at inducement. What changes actually do occur create a new set of actor Positions, on which the forecasting process is again applied. Though KTAB does support simulating this complex bargaining process, those capabilities will not be discussed further in this paper.

This paper will focus on the second category of CDMP. In the cases examined here, there is a simple algorithm, or "forecasting rule", to describe the end result of the negotiation process. In general, as in Black (1948), the forecast is that the CDMP will result in that outcome which the whole group prefers to all other outcomes i.e. the Condorcet winner.

It is important to bear in mind that the forecasting rule is not chosen arbitrarily but derived from a concept of how the underlying dynamic negotiation works. While the forecasting rule picks out a plausible final outcome, the underlying dynamic process still takes place.

In complex cases where the forecasting rule is very difficult to apply, it is sometimes more efficient to simply simulate the negotiation process in full. KTAB provides utilities to build such simulations, though they will not be discussed further in this paper. We shall address them in future papers.

In either static choices or dynamic negotiations, an actor's willingness to promote or oppose a particular outcome will be a function of that outcome's desirability to the actor and the desirability of alternatives. The desirability of an outcome can informally be viewed as utility; we will use the term "Attractiveness" to cover both. In the one-dimensional case, the Attractiveness to an actor of various Positions along the line can easily be graphed as a curve. However, even in the one-dimensional, simple dynamic CDMP, there are several possible complications that can arise.

- Different actors may also have different amounts of power to sway the collective decision making process. For instance, if Y is X's boss X can only advise and influence – Y ultimately calls the shots.
- Equally, the various actors may not all have the same level of interest in the outcome. Consider if Y is the department head and does not manage the day to day decisions, delegating those to X. Actor Y may ultimately decide, but Actor X likely cares about the decision far more than Y does and will lobby hard to influence Y's decision.
- Most severely, while each actor's Attractiveness curve will be peaked at their preferred Position, the Attractiveness curves need not have a single peak: actors may have a high preference for two different options and a lower preference for intermediates.

The models to analyze these types of negotiation are often referred to as generalized voting models. In this context voting is just the exercise of Influence (Bueno de Mesquita, 1990). It does not necessarily imply formal voting and certainly does not demand a one person, one vote approach. Votes need not be cast in the ballot boxes of democracies. They are more often nods of the head in a meeting of a company's senior managers, expenditure of resources to advertise pros and cons of policies, or invisible indications of assent as coalitions are built. For many national policy issues, an actor could represent a profession, an industry, an ethnic group, or similar informal collection with common interests. More powerful actors (be they individuals or groups of individuals) may have more votes, even if they do not necessarily bother to use them all.

When we view voting as simply the exercise of Influence we can see that such models have extensive applicability. Understanding these and other subtleties takes the theory in varied technical directions, which may be the subject of subsequent papers.

As we proceed through the next sections, the models we present will progressively become more nuanced, taking into account an actor's Influence, how much they care and even their attitude toward compromise and risk, alongside their Position. Each concept will be introduced, both descriptively and then mathematically, in a step-by-step illustration of KTAB's applicability and its theoretical underpinnings.

Generalized voting is the exertion of Influence to promote or resist a possible outcome. The method to estimate how much of an actor's exercised power will be exerted between different choices is termed a "voting model". For the purposes of this paper, we will examine two particular voting models. This is largely because they yield particularly simple forecasting rules for the outcome of non-coercive negotiation.

4.3 The Median Voter Theorem

The Median Voter Theorem (MVT) is one of the most celebrated results in analytical political science; it was originally derived and presented in Black (1948). The MVT makes several critical assumptions about the structure of a one-dimensional model.

- The Positions of actors can be represented by points on a PSPP, and all actors have the same understanding of the PSPP and of Positions.
- Actors exercise their entire voting "weight" when confronted with a choice between two alternative Positions, no matter how large or small the difference is between the alternatives. Because the actors exert their full weight either one way or the other, we term this "binary voting". This is a very plausible assumption in the context of committee voting, where the only way for an individual legislator to exert Influence is to vote for or against a proposal. Different ideas of "weight" give different results; we will explore this issue in greater detail later.

– The attractiveness of options to an actor is strictly declining away from the actor's preferred Position. This is equivalent to requiring that the actor's attractiveness function has a single peak at the preferred Position.

On a horizontal PSPP, the Weighted Median Position (WMP) is defined to be the Position which has at least half the weight at that Position or to its left, as well as having at least half the weight at that Position or to its right. The MVT then says that the WMP will be the outcome of the CDMP.

The logic of the MVT is essentially geometric; hence the underlying model of negotiation is often called the "spatial model of politics". It is best explained by imagining how a dynamic negotiation would end, then considering how the negotiation will approach that end. It is essential to the logic that Positions can be represented on a one-dimensional spectrum; the MVT does not hold in multiple dimensions except under extremely artificial assumptions.

Given that actors exert all their weight for the more attractive of two options, and attractiveness declines with distance, the detailed shape of an actor's attractiveness function hardly matters : the actor will use their full weight to promote whichever option is closer to their own Position. An attractiveness function must always exist (or else it could not have the required properties), but in this model we need not consider the exact shape of the function, only the distance between options.

First, consider how the whole group of actors would choose between the WMP and any alternative Position – say X. Suppose that the alternative X is to the right of the WMP on a horizontal PSPP. At least half the voting weight is to the left of the WMP (by definition), and all those voters will prefer the WMP over X. Of those actors whose Positions are between the WMP and the X, some will be closer to the WMP and will prefer it. Hence, the whole group will take the WMP over any alternative on the right (see Figure 4.1). An exactly similar argument shows that the group will choose the WMP over any alternative on the left. Hence, the WMP is that outcome which is preferred over all others i.e. the Condorcet Winner. Once it is proposed by any actor, no other Position on the PSPP can possibly displace it, and no further negotiation will occur because all the actors know that it would be pointless. This situation can be expected to persist until something changes the balance of Influence.

Second, as long as the whole group is not yet at the WMP, actors always have an incentive to make proposals closer to the WMP. Suppose that the group currently favors some proposed outcome Y, which lies to the right of the WMP. Consider another Position, Z, between the WMP and Y. Each actor at or to the left of Z (on both sides of the WMP, as long as they are to left of Z) would individually prefer Z over Y, and hence would have an incentive to propose Z. By essentially the same logic as before, the group as a whole would also prefer Z over Y, so the proposal would be strategically sound. In this manner, the dynamic process of repeated strategic proposals, each chosen to win enough support to prevail while advancing the proposer's goals, would inevitably bring the group to the WMP, at which point the CDMP terminates. See panels B and C of Figure 4.1.

One of the extremely useful features of the MVT is that it provides a very simple forecasting rule for the outcome of the CDMP. Rather than simulate the iterative process of strategically formulating and voting on a series of proposals, one need only scan across the PSPP to find the WMP, where the balance of Influence will produce a stable outcome.

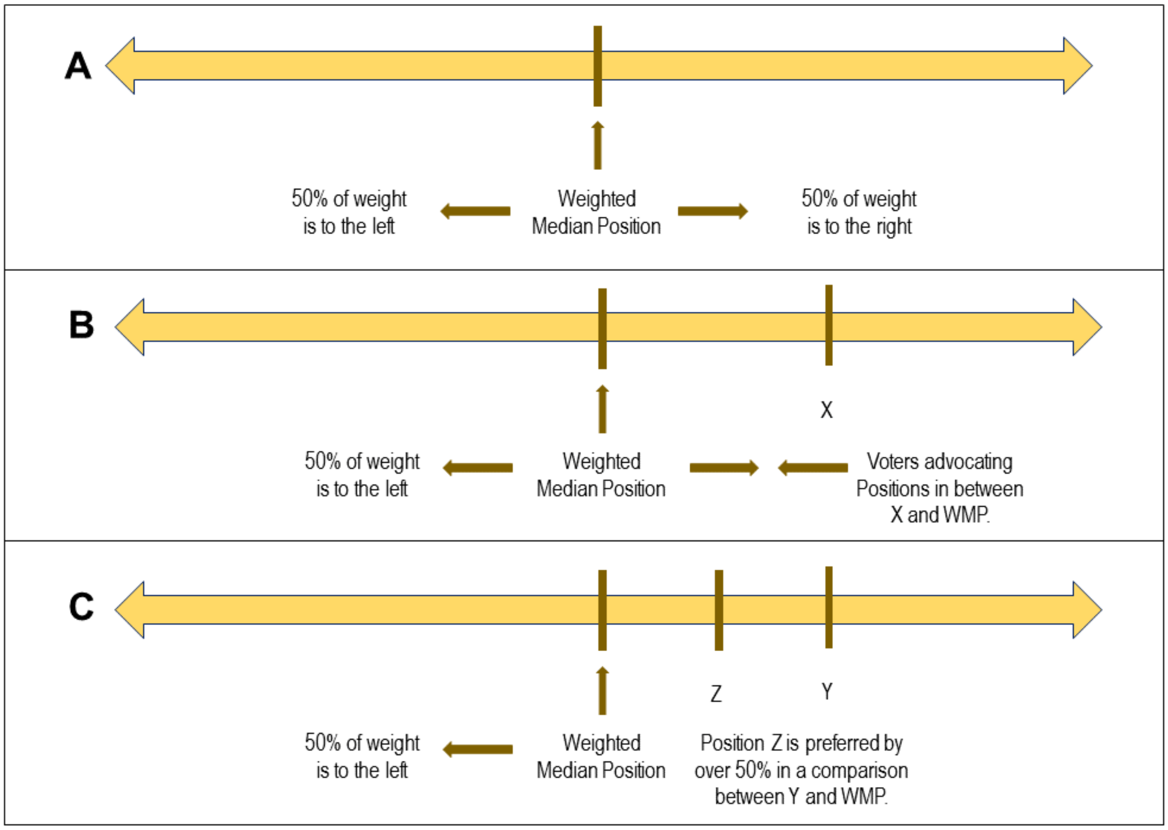

Figure 4.1. A: Illustrating the Weighted Median Position (WMP). This is the Position where at least half the vote (or "weight") is to the left and half to the right. **B:** Imagine a Position to one side of the WMP, here labelled X. Some of the voters between X and the WMP will favor X, some the WMP. Because we know that at least 50% of the vote is at or to the left of the WMP, at least 50% automatically prefers the WMP to X. Therefore, any additional support for the WMP (from those in between X and the WMP but closer to the WMP than to X) will add their support to the existing 50% to give a group majority in favor of the WMP over X. This is true for any Position other than the WMP. Therefore, the WMP will be preferred by the whole group to all other Positions. **C:** Just as the WMP is preferred over any non-median Position, so a non-median Position closer to the WMP is preferred over any other non-median Position.
Source: KAPSARC

4.4 The Central Position Theorem

Outside committee voting, there are many situations in which actors would be expected to exert their Influence in a more nuanced fashion. The simplest model of such behavior is that, when confronted with a choice between two options that have difference Attractiveness, the actor's response will depend directly on the difference in Attractiveness: the level of Influence exerted is proportional to the stakes involved. We call this proportional voting. Note that if both options have high attractiveness, then the stakes are low. The stakes are high only when the difference in attractiveness between the outcomes is high. Proportional voting violates the requirements of the MVT, which requires binary voting, so the question arises as to when, if ever, proportional voting can have a Condorcet Winner.

To address this kind of situation, the Central Position Theorem (CPT) was derived in Wise (2010a, 2010b) and published in Jesse (2011) . It alters just one assumption of the MVT in order to derive a different forecasting rule. The key condition of the MVT is the assumption that actors exercise their entire voting "weight" when confronted with a choice between two alternative Positions, no matter how large or small is the difference between the alternatives. The CPT alters this assumption, while dropping all the others.

Proportional voting is not applicable to standard legislative voting, because legislators can only vote “yea” or “nay”, without the possibility of fractional votes. Further, because the act of casting a vote is costless per se, there is no reason for a large voting block to do anything less than have all its members cast their votes the same way, if the leadership judges that the net political costs and benefits warrant doing so. It is important to distinguish between the political costs and benefits of different outcomes (which is what analysts usually mean when they describe a legislative vote as damaging or beneficial) and the negligible effort of actually registering a vote. The political costs and benefits of a legislative voting one way or another may be estimated by a complex sub-model or by simple proximity on a line (as in the models we will introduce in section 5), but again the distinction here is that while the political consequences of voting (or making someone vote) may be damaging or beneficial, the simple physical act of saying “yea” or “nay”, of exerting Influence through voting per se, is costless.

However, if the actor represents an interest group which exerts Influence through advertising campaigns in elections, or a nation state which exerts Influence by deploying its military, then exertion of effort roughly proportional to the stakes is often observed. Little effort is made when the stakes are low; great efforts are made when the stakes are high. This is because the actor must pay a cost to exert Influence, either directly in terms of dollars that are no longer available for other advertising campaigns, or in terms of military risks incurred by lessened deterrence in other areas. Great efforts incur great costs, and hence will not be undertaken for small gains.

The CPT states that under proportional voting, the CDMP will lead to an outcome which maximizes a function we call the Weighted Attractiveness Score (WAS). This will be explained in more detail in section 5, when we also introduce the necessary mathematical calculations. That outcome is called the Central Position, and it is the outcome which all the actors will prefer to any other outcome. In other words, it is the Condorcet Winner under proportional voting. By almost the same logic as the MVT, actors will always have an incentive to propose options with higher WAS, until the Central Position is proposed. At that point, no other option can displace the Central Position and no further negotiation will occur, because every actor knows that any further proposals will be rejected. This situation can be expected to persist until something changes the balance of Influence.

With binary voting, the detailed shape of an actor’s Attractiveness curve is irrelevant: actors exert their full voting weight in favor of whichever alternative is closer to their own Position. With proportional voting, the shape of the Attractiveness curve is necessary to compare the attractiveness of two options and thus gauge how much Influence an actor will exert for one or the other.

The logic of the CPT is fundamentally non-spatial, which makes it very broadly useful but also harder to visualize. The strong assumptions which made the MVT easy to visualize are no longer used, so there is no simple geometric explanation of the CPT. This is intimately connected to the fact that the CPT applies not only to one-dimensional or multi-dimensional problems but also to situations in which notions of distance are not very useful. However, the proof of the CPT is quite simple algebraically; it can be found in section 6. What is important for our purposes here is that it can easily be used in one-dimensional spatial CDMPs.

One of the extremely useful features of the CPT is that it provides a simple forecasting rule for the outcome of the CDMP. Rather than simulate the iterative process of strategically formulating proposals and generalized voting on them, one can find the option which maximizes the Weighted Attractiveness Score, where the balance of Influence will produce a stable outcome.

5. Exploring one-dimensional spatial models

5.1 Section summary

Two different voting rules are explored, via four example models. The effects of alternative parameterizations are compared. One comparison contrasts giving powerful actors a weight greater than or equal to weak actors. Another alternative compares the effect of actors' willingness to compromise and accept risks (or not); the representation of this difference in terms of utility models is explained. Step-by-step instructions for the calculations of each example are provided.

KTAB allows the construction of many different models; in this paper we present just four. These four should not be seen as indicative of KTAB's range, rather they make an understandable introduction to the underlying concepts and build on each other incrementally. In order to keep the focus on the models and underlying theory, we will, as stated before, use the long published data from Bueno de Mesquita's 1984 paper on the Iranian economy.

With each model we proceed stepwise through its theory, underlying logic, and basic algebra. The results from each model are compared to Bueno de Mesquita's. The four models give three different answers; only one agrees with the original paper. The point is that different models with different forecasting rules give different answers. What is right or wrong, what wins or loses, depends upon the analyst's view of the type of CDMP being investigated and how it should be modeled.

Should votes be binary or proportional? Should all actors be modeled as equally powerful (one man, one vote) or with different levels of power (as in shareholder meetings)? Should the risk attitude (the propensity to compromise) of actors be included? The answers to these questions depend on the type of CDMP and determine the most suitable model.

5.2 Introducing the basic one-dimensional models

This section develops and discusses four basic models that come prebuilt in KTAB and which can be used to place bounds on a range of plausible outcomes which a whole group of actors might choose when engaged in a one-dimensional CDMP. They will be illustrated with the economic policy issue presented in section 2.4.

While an overview was given in Bueno de Mesquita (1984a), no technical description was given of the forecasting process, the coercive bargaining process, or the shape of attractiveness curves. Fortunately, a thorough third-party analysis of several papers and books related to Bueno de Mesquita (1984a) can be found in Scholz et al. (2011; e.g. Bueno de Mesquita 1997, 2002, Bueno de Mesquita and Lalman 1986, 1992, Feder, 2002). Scholz et al. (2011) assembles a detailed, explicit, and mathematically coherent view of the model apparently used in Bueno de Mesquita (1984a). The first economic analysis apparently uses simple dynamic negotiation to forecast an outcome. The second economic analysis apparently uses a complex dynamic bargaining process to estimate changes in actor's preferences, and then uses the same simple dynamic negotiation to forecast a new outcome given those new preferences. In this paper, we will focus on the simple dynamic bargaining of the second example. Nonetheless, to repeat, we are not here trying to recreate Bueno de Mesquita's (1984a) analysis or the algorithms of Scholz et al. (2011); we are merely using that example (and its data) for expediency and clarity.

5.3 Basic data

Just as explained in section 3, Bueno de Mesquita (1984a) assigns numeric values for Position, Influence, and Salience. In line with our explanation of section 3, and as introduced in section 2.4, Bueno de Mesquita (1984a) sets out the actors and their associated Positions using a PSPP; for the sake of clarity Figure 5.1 reproduces the PSPP from Bueno de Mesquita (1984a), as shown in figure 2.1 of section 2.4, this time with his forecast winner highlighted. The Positions of the actors are arrayed on a phrase-anchored PSPP entitled "Issue: What is the attitude of each group toward the role of the government in the economy?" The phrase "Complete Free Market" anchors one end of the PSPP and "Complete Government Control" anchors the other.

Figure 5.2 reproduces the Salience scale from Bueno de Mesquita (1984a). Note that his original gives the Salience for all 27 actors from both analyses in the paper. We present only the 15 actors included in the paper's second analysis upon which we have based our explanations.

Finally, Bueno de Mesquita (1984a) states that the Influence scores in that paper's Table 1 were quantified by obtaining expert estimates (consistent with section 3.2) of Influence, and normalizing so that all his actors sum to 100; the individual scores range from 0 to 12.4. The exact underlying scale does not matter (remember that in section 3.5.2 we set out a scheme for Influence to be measured on a 0-100 scale) and can be taken as arbitrary for present purposes. What is important is the actor's relative score within the scale.

All the data from Bueno de Mesquita (1984a) are brought together in our Table 5.1, and constitute the input for our models to follow. In Table 5.1, column A enumerates a rank ordering of the Positions (column C), lowest to highest. We will also use these numbers as indexes to indicate the associated actors in column B. The actors' Position and Salience

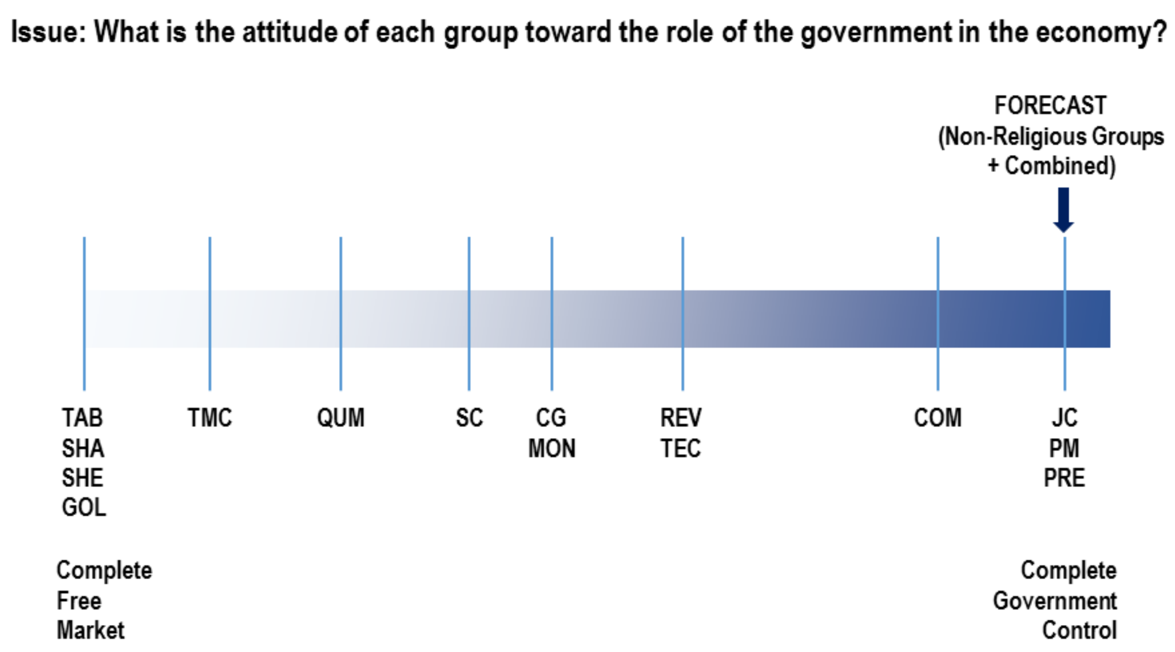

Figure 5.1: We reproduce Bueno de Mesquita's representation of the actors involved in his investigation of economic policymaking in Iran following the 1978 revolution. He poses a simple question: "What is the attitude of each group toward the role of the government in the economy?". The various attitudes of the actors are then arrayed along a line. The relative distances along the spectrum from "Complete Free Market" to "Complete Government Control" can be turned into a set of distances running 0 to 1, with "Complete Free Market" set to 0 and "Complete Government Control" set to 1. This is the same spectrum of positions as shown in Figure 2.1.
Source: Bueno de Mesquita (1984a)

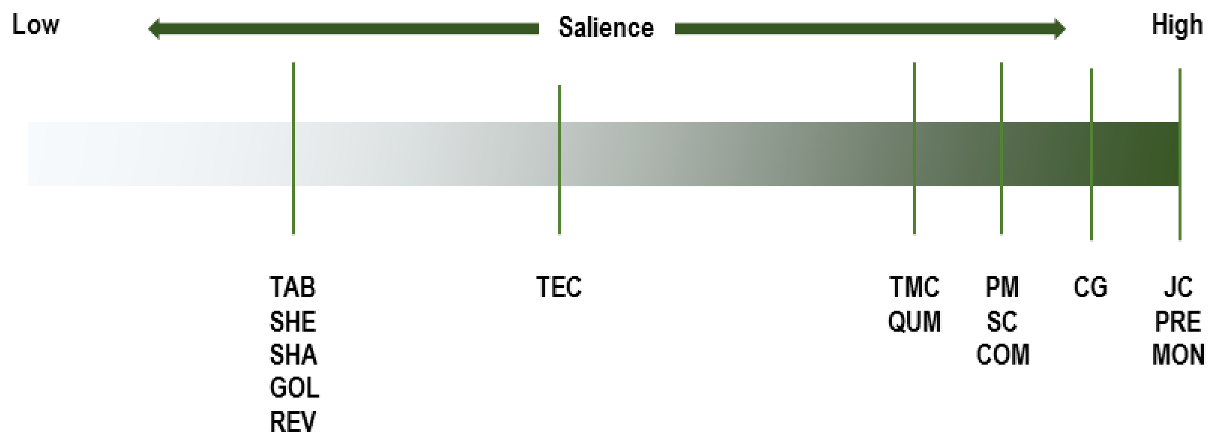

Figure 5.2: We reproduce Bueno de Mesquita's representation of the Salience of the actors, but modify it to show only the 15 actors included in his second economic analysis. The Salience of the various actors can be read from the scale where TAB and SHE, amongst others, are at 0.31 and JC, PRE and MON are at 1 (or 100). Remember that a Salience of 0 indicates that the actors have no interest in the issue at all; a Salience of 1 means that the issue is of the utmost importance.
Source: Bueno de Mesquita (1984a)

A	B	C	D	E
Order	Agent	Position	Salience	Infuence
1	TAB	0.00	0.31	0.0
2	SHA	0.00	0.31	0.6
3	SHE	0.00	0.31	0.0
4	GOL	0.00	0.31	0.6
5	TMC	0.13	0.79	3.4
6	QUM	0.26	0.79	4.5
7	SC	0.39	0.86	4.5
8	CG	0.48	0.93	9.0
9	MON	0.48	1.00	0.1
10	REV	0.60	0.31	12.4
11	TEC	0.60	0.52	0.6
12	COM	0.87	0.86	11.8
13	PM	1.00	1.00	9.0
14	PRE	1.00	1.00	10.7
15	JC	1.00	1.00	11.3

Table 5.1: This is the data that we will use as the basis for our exploration of one-dimensional spatial models using KTAB. Only the 15 actors involved in the second economic analysis are listed here, rank ordered by Position.
Source: Bueno de Mesquita (1984a)

values are indicated in Bueno de Mesquita (1984a) by tick marks on non-numeric scales. We estimated the actual values by direct measurement on the corresponding diagrams, then normalizing them onto a zero-one range.

5.4 Building and solving the models

Bueno de Mesquita (1984a) forecasted a possible outcome as a single Position on the line (1.0, the Position of PM, PRE, and JC), with the intent of informing discussion. Unfortunately, it did so without providing the underlying model. In what follows, we illustrate this one-dimensional deterministic application of KTAB by applying four different CDMP models to the data from the second economic issue presented in Bueno de Mesquita (1984a).

5.4.1 Model 1: Weighted Median Position of Exercised Powers

This model is based on the Median Voter Theorem (MVT) as presented in Black (1948). In this particular example, we use the Exercised Power scores of individual as the indication of voting weight.

Thus, in this first model we find the WMP and forecast it as the outcome: COM at 0.87 (contrast this to Bueno de Mesquita's (1984a) prediction of PM, PRE and JC at 1.0). See Table 5.2 for the worked example.

As explained earlier, Exercised Power represents the amount of power the actor will bring to bear in the negotiation. We define e_i, the Exercised Power of actor i in swaying the results of the bargaining as $e_i = S_i \times I_i$, that is as the product of the actor's Salience and Influence (Columns D and E of Table 5.1). Table 5.2 displays each actor's Exercised Power in column D. The total exercised power appears in column E; it is shown per Position, not by actor, because it is the Positions that are attractive, not actors. Column F presents the cumulative sums of the Exercised Power values in column E, starting at the top of the table (far left of the PSPP) with ID=1, actor TAB. Column G presents the accumulated sum starting at the bottom of the table (far right of the PSPP) with ID=15, actor JC. Thus, the entries in column F show total Exercised Power at or to the left of each Position, while column G shows the total Exercised Power at or to the right of each Position. The total Exercised Power is 64.27, so the threshold for the WMP is half that, 32.14. By inspection, we see that the only Position which has 32.14 or more in both columns F and G is Position 0.87, occupied by COM. This is indicated by a check mark in column H. This Position is the model's declared (Condorcet) winner.

5.4.2 Model 2: Equally Weighted Proportional Voting

The implicit theory of Model 1 was that the outcome of the collective decision process would be strongly Influenced by the disparity in the Exercised Powers of the actors. Actors have different amounts of Influence, and each uses a constant level of Exercised Power between any two options.

Model 2 differs from the previous model in two ways:

- All actors are assigned equal weight to try to approximate a more egalitarian collective judgment

- Actors use proportional voting, so they exert differing levels of Influence to promote outcomes based on how high or low the stakes are to them

In Model 1, the only important characteristic of the attractiveness function was that it be single peaked i.e. one option was preferred to all others and more distant options were less attractive; the exact shape did not matter. However, in Models 2, 3, and 4, actors calibrate the Influence they exert by the perceived

A	B	C	D	E	F	G	H
Order	Actors	Position	Exercised Power by Actor	Exercised Power at Position	Sum at or to Left of Position	Sum at or to Right of Position	Forecast Rule
ID	$\mathcal{A}$	θ_i	e_i		$\sum_{i=1}^{ID} e_i$	$\sum_{i=ID}^{15} e_i$	Weighted Median
1	TAB	0.00	0.00	0.37	0.37	64.27	
2	SHA	0.00	0.18				
3	SHE	0.00	0.00				
4	GOL	0.00	0.18				
5	TMC	0.13	2.69	2.69	3.06	63.90	
6	QUM	0.26	3.57	3.57	6.63	61.21	
7	SC	0.39	3.88	3.88	10.51	57.64	
8	CG	0.48	8.38	8.48	18.98	53.77	
9	MON	0.48	0.10				
10	REV	0.60	3.82	4.12	23.11	45.29	
11	TEC	0.60	0.31				
12	COM	0.87	10.17	10.17	33.27	41.17	✓
13	PM	1.00	9.00	31.00	64.27	31.00	
14	PRE	1.00	10.70				
15	JC	1.00	11.30				

Table 5.2: Results for Model 1. The actors, their Positions and Exercised Powers are listed in columns B, C and D. Column E shows the Exercised Power accrued to each Position from the various actors advocating that Position. Column F then sums the Exercised Power at each Position from top to bottom; column G, from bottom to top. The WMP is the Position that has 50% or more of the weight on either side. Note that the sum of all Exercised Power is 64.27; half of that is 32.14. To find the WMP we identify a Position that has 32.14 or more on either side of it. This is Position 0.87, held by actor 12, COM. (In Bueno de Mesquita's analysis, Position 1 is forecast to be the winner. It is likely, therefore, that he was using a different model.) The shading of rows is merely to aid the eye in distinguishing between Positions as opposed to actors.
Source: KAPSARC

stakes, so the precise shape, and therefore the numerical values, of the attractiveness function must be considered. We now introduce the additional notation necessary to handle shape information. The Attractiveness to actor i of a normalized (0 to 1) Position is symbolized as $A_i(\Theta)$: it is actor i's score for possible outcome Θ. The attractiveness score (where all actors carry the same weight of 1) of the j-th Position is the sum over all actors:

$$\omega_j = \sum_{i=1}^{N} A_i(\theta_j)$$

The Attractiveness of any possible outcome to an actor is determined by the distance between that outcome and the actor's Position. Since the actor's Position is its advocated point, every other Position has a lower Attractiveness. In Model 2, we formally define the Attractiveness A_i to decline linearly with distance d_i between the actor's Position Θ_i and any other Position Θ:

$$d_i(\theta) = |\theta_i - \theta|$$

$$A_i(\theta) = 1 - d_i(\theta)$$

Both the actor's Position, Θ_i, and the alternative, Θ, are on the same zero to one scale as in column C of Table 5.1. Note that in other spatial models the Attractiveness may be defined in other ways so that it does not need to decline linearly with distance; KTAB also supports building models in which Attractiveness does not depend on any measure of distance. For example, the relative Attractiveness of two tax policies might depend on their differing economic consequences, so that assessing Attractiveness requires an economic model (crucially, different actors might expect different consequences) rather than a simple measure of distance.

In this model, the Attractiveness values are added up for the Position, across every actor, and the maximum Attractiveness Score is used to forecast the outcome of the CDMP. Table 5.3 displays the key data and results.

The Attractiveness Score of a Position is calculated by adding the Attractiveness of the Position to each Actor. This is best done using a matrix. We will now illustrate how the Attractiveness Score is calculated by looking at the calculations that go into the Attractiveness Score (10.14) of the Position of SC, actor 7.

Let us start by calculating the distance to TMC, actor 5, of the Position advocated by SC. The highlighted row of Table 5.4 is the row corresponding to i=5; the highlighted column of table 5.4 is the column corresponding to j=7.

The Position of TMC is Θ_5 = 0.13, the Position in the fifth row.

The Position of SC is Θ_7 = 0.39, the Position in the seventh column.

The distance between them is $d_5(\Theta_7)$ = | 0.13 - 0.39 | = 0.26. Table 5.4 recreates the complete intermediate matrix of distance scores for our data set.

The next step is to move from distance scores to Attractiveness scores. The Attractiveness to TMC of the Position held by SC is $A_5(\Theta_7) = 1 - d_5(\Theta_7)$ = 1 - 0.26 = 0.74. This is the number in bold at the intersection of the fifth row and the seventh column of Table 5.5.

This process can be repeated for every entry in the seventh column, giving all the highlighted numbers on the seventh column. The Attractiveness score for the Position of SC, in the seventh column, is the sum of the Attractiveness values in the seventh column, which is 10.14. This is the number seen in Table 5.3.

A	B	C	D	E
Order	Actors	Position	Attractiveness Score	Forecast Rule
ID	$\mathcal{A}$	θ_i	$\sum_{i=1}^{15} A_i(\theta_j)$	Maximum Attractiveness Score
1	TAB	0.00	8.19	
2	SHA			
3	SHE			
4	GOL			
5	TMC	0.13	9.11	
6	QUM	0.26	9.77	
7	SC	0.39	10.14	
8	CG	0.48	10.23	✓
9	MON			
10	REV	0.60	9.85	
11	TEC			
12	COM	0.87	8.00	
13	PM	1.00	6.81	
14	PRE			
15	JC			

Table 5.3: Results for Model 2. The actors, their Positions and Attractiveness Scores are listed in columns B, C and D. Tables 5.4 and 5.5 illustrate how to calculate the Attractiveness Score. Briefly it is the sum of a Position's attractiveness in the eyes of each actor. Under this formulation, the Position (0.48) of actors 8 and 9, (CG and MON), has the highest Attractiveness Score. This is a much more moderate Position than forecast by Model 1.
Source: KAPSARC

			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
			TAB	SHA	SHE	GOL	TMC	QUM	SC	CG	MON	REV	TEC	COM	PM	PRE	JC
			0.00	0.00	0.00	0.00	0.13	0.26	0.39	0.48	0.48	0.60	0.60	0.87	1.00	1.00	1.00
1	TAB	0.00	0.00	0.00	0.00	0.00	0.13	0.26	0.39	0.48	0.48	0.60	0.60	0.87	1.00	1.00	1.00
2	SHA	0.00	0.00	0.00	0.00	0.00	0.13	0.26	0.39	0.48	0.48	0.60	0.60	0.87	1.00	1.00	1.00
3	SHE	0.00	0.00	0.00	0.00	0.00	0.13	0.26	0.39	0.48	0.48	0.60	0.60	0.87	1.00	1.00	1.00
4	GOL	0.00	0.00	0.00	0.00	0.00	0.13	0.26	0.39	0.48	0.48	0.60	0.60	0.87	1.00	1.00	1.00
5	TMC	0.13	0.13	0.13	0.13	0.13	0.00	0.13	0.26	0.35	0.35	0.47	0.47	0.74	0.87	0.87	0.87
6	QUM	0.26	0.26	0.26	0.26	0.26	0.13	0.00	0.13	0.21	0.21	0.34	0.34	0.61	0.74	0.74	0.74
7	SC	0.39	0.39	0.39	0.39	0.39	0.26	0.13	0.00	0.09	0.09	0.22	0.22	0.48	0.61	0.61	0.61
8	CG	0.48	0.48	0.48	0.48	0.48	0.35	0.21	0.09	0.00	0.00	0.13	0.13	0.39	0.52	0.52	0.52
9	MON	0.48	0.48	0.48	0.48	0.48	0.35	0.21	0.09	0.00	0.00	0.13	0.13	0.39	0.52	0.52	0.52
10	REV	0.60	0.60	0.60	0.60	0.60	0.47	0.34	0.22	0.13	0.13	0.00	0.00	0.26	0.40	0.40	0.40
11	TEC	0.60	0.60	0.60	0.60	0.60	0.47	0.34	0.22	0.13	0.13	0.00	0.00	0.26	0.40	0.40	0.40
12	COM	0.87	0.87	0.87	0.87	0.87	0.74	0.61	0.48	0.39	0.39	0.26	0.26	0.00	0.13	0.13	0.13
13	PM	1.00	1.00	1.00	1.00	1.00	0.87	0.74	0.61	0.52	0.52	0.40	0.40	0.13	0.00	0.00	0.00
14	PRE	1.00	1.00	1.00	1.00	1.00	0.87	0.74	0.61	0.52	0.52	0.40	0.40	0.13	0.00	0.00	0.00
15	JC	1.00	1.00	1.00	1.00	1.00	0.87	0.74	0.61	0.52	0.52	0.40	0.40	0.13	0.00	0.00	0.00

Table 5.4: The intermediate matrix of distance values, $d_i\ (\Theta)\ =\ |\Theta_i - \Theta|$. The column shading serves merely to highlight an example Position (Θ = 0.39, actor SC) against which we are calculating the distances. The rows shows the actor against which the alternative Position is being compared; row 5 has been shaded for illustration.
Source: KAPSARC

			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
			TAB	SHA	SHE	GOL	TMC	QUM	SC	<u>CG</u>	<u>MON</u>	REV	TEC	COM	PM	PRE	JC
			0.00	*0.00*	*0.00*	*0.00*	*0.13*	*0.26*	*0.39*	*<u>0.48</u>*	*<u>0.48</u>*	*0.60*	*0.60*	*0.87*	*1.00*	*1.00*	*1.00*
1	TAB	0.00	1.00	1.00	1.00	1.00	0.87	0.74	0.61	0.52	0.52	0.40	0.40	0.13	0.00	0.00	0.00
2	SHA	0.00	1.00	1.00	1.00	1.00	0.87	0.74	0.61	0.52	0.52	0.40	0.40	0.13	0.00	0.00	0.00
3	SHE	0.00	1.00	1.00	1.00	1.00	0.87	0.74	0.61	0.52	0.52	0.40	0.40	0.13	0.00	0.00	0.00
4	GOL	0.00	1.00	1.00	1.00	1.00	0.87	0.74	0.61	0.52	0.52	0.40	0.40	0.13	0.00	0.00	0.00
5	TMC	0.13	0.87	0.87	0.87	0.87	1.00	0.87	**0.74**	0.66	0.66	0.53	0.53	0.26	0.13	0.13	0.13
6	QUM	0.26	0.74	0.74	0.74	0.74	0.87	1.00	0.88	0.79	0.79	0.66	0.66	0.40	0.26	0.26	0.26
7	SC	0.39	0.61	0.61	0.61	0.61	0.74	0.88	1.00	0.91	0.91	0.78	0.78	0.52	0.39	0.39	0.39
8	CG	0.48	0.52	0.52	0.52	0.52	0.66	0.79	0.91	1.00	1.00	0.87	0.87	0.61	0.48	0.48	0.48
9	MON	0.48	0.52	0.52	0.52	0.52	0.66	0.79	0.91	1.00	1.00	0.87	0.87	0.61	0.48	0.48	0.48
10	REV	0.60	0.40	0.40	0.40	0.40	0.53	0.66	0.78	0.87	0.87	1.00	1.00	0.74	0.60	0.60	0.60
11	TEC	0.60	0.40	0.40	0.40	0.40	0.53	0.66	0.78	0.87	0.87	1.00	1.00	0.74	0.60	0.60	0.60
12	COM	0.87	0.13	0.13	0.13	0.13	0.26	0.40	0.52	0.61	0.61	0.74	0.74	1.00	0.87	0.87	0.87
13	PM	1.00	0.00	0.00	0.00	0.00	0.13	0.26	0.39	0.48	0.48	0.60	0.60	0.87	1.00	1.00	1.00
14	PRE	1.00	0.00	0.00	0.00	0.00	0.13	0.26	0.39	0.48	0.48	0.60	0.60	0.87	1.00	1.00	1.00
15	JC	1.00	0.00	0.00	0.00	0.00	0.13	0.26	0.39	0.48	0.48	0.60	0.60	0.87	1.00	1.00	1.00
Column Sum			8.19	8.19	8.19	8.19	9.11	9.77	10.14	<u>10.23</u>	<u>10.23</u>	9.85	9.85	8.00	6.81	6.81	6.81

Table 5.5: The matrix of Attractiveness values and the Attractiveness Scores. The bottom row, labelled Column Sum, is what is shown in column D of Table 5.3. Actors 8 and 9 (CG and MON) are underlined, showing the highest Attractiveness Scores. The Attractiveness values are calculated using the distance values in Table 5.4 with the following equation: $A_i(\Theta) = 1 - d_i(\Theta)$
Source: KAPSARC

This process can be repeated for every column, giving the row of column sums, one for each Position. The highest of the column sums is 10.23. Because we have used a different model, with a different weighting and different voting rule, we obtain a different forecast : Position 0.48 occupied by both CG and MON, which are underlined in Table 5.5. Again, note that this also differs from Bueno de Mesquita's forecast outcome.

5.4.3 Model 3: Unequally Weighted Proportional Voting

Model 3 differs from Model 2 in only one way:· The actors have different weights from each other. (Thus, Model 2 is a special case of Model 3.)

It allows for this by calculating for each Position the sum, across all actors, of the products of each actor's Exercised Power score and its Attractiveness Score. The Weighted Attractiveness Score (WAS) of the *j*-th Position is the sum, across all actors, of the products of each actor's Exercised Power score and its Attractiveness Score:

$$\omega_j = \sum_{i=1}^{N} e_i A_i(\theta_j)$$

Table 5.6 displays the key data and results; the reader is urged to compare it to Table 5.3. Column D of Table 5.6 contains the WAS for the several Positions.

Once again, the WAS, weighted by Exercised Power, is best calculated through a matrix. The matrix $e_i A_i(\Theta_j)$ for this example is given in Table 5.7. We now illustrate the calculations that go into the WAS (37.46) for the Position of SC, actor 7. Let us start by calculating the Attractiveness to TMC, actor 5, of the Position advocated by SC. The highlighted row of Table 5.7 is the row corresponding to (actor) i=5; the highlighted column of Table 5.7 is the column corresponding to (Position) j=7.

The Position of TMC is Θ_5 = 0.13, the Position in the fifth row.

The Position of SC is Θ_7 = 0.39, the Position in the seventh column.

The distance between them is $d_5(\Theta_7)$ = | 0.13 - 0.39 | = 0.26. This is the number in bold at the intersection of the fifth row and the seventh column of Table 5.4; so far the calculation is exactly identical to that in Model 2.

The attractiveness to TMC of the Position held by SC is $A_5(\Theta_7) = 1 - d_5(\Theta_7) = 1 - 0.26 = 0.74$. This is the number in bold at the intersection of the fifth row and the seventh column of Table 5.5; again so far the calculation is exactly identical to that in Model 2.

The Exercised Power of actor TMC is e_5 = 2.69 of, as per column D of Table 5.2.

The Weighted Attractiveness is 2.69 x 0.74 = 2.00; this is the bold number at the intersection of the fifth row and seventh column of Table 5.7.

This process can be repeated for every entry in the seventh column, giving all the highlighted numbers in the seventh column. The WAS for the seventh column is the sum of the Attractiveness Values on the seventh column, which is 37.46.

This process can be repeated for every column, giving the row of column sums, one for each Position. Because we have used a different model, with a different weighting and different voting rule, we obtain a different forecast: Position 0.87 occupied by COM, which is underlined in Table 5.7.

A	B	C	D	E
Order	Actor	Position	Weighted Attractiveness Score	Forecast Rule
ID	A	θ_i	$\sum_{i=1}^{15} e_i A_i(\theta_j)$	Maximum WAS
1	TAB	0.00	15.15	
2	SHA			
3	SHE			
4	GOL			
5	TMC	0.13	23.47	
6	QUM	0.26	31.09	
7	SC	0.39	37.46	
8	CG	0.48	41.31	
9	MON			
10	REV	0.60	44.68	
11	TEC			
12	COM	0.87	49.43	✓
13	PM	1.00	49.13	
14	PRE			
15	JC			

Table 5.6: Results for Model 3. The actors, their Positions and Weighted Attractiveness Scores (WAS) are listed in columns B, C and D. In this model, the Attractiveness values that were calculated for Model 2 are weighted by the actor's Exercised Power. The full matrix of Weighted Attractiveness values is shown in Table 5.7, which may be counted as the third in the series following Tables 5.4 and 5.5. Under this formulation, actor 12, COM, has the highest WAS. This is the same outcome as for Model 1.
Source: KAPSARC

			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
			TAB	SHA	SHE	GOL	TMC	QUM	SC	CG	MON	REV	TEC	COM	PM	PRE	JC
			0.00	0.00	0.00	0.00	0.13	0.26	0.39	0.48	0.48	0.60	0.60	0.87	1.00	1.00	1.00
1	TAB	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	SHA	0.00	0.18	0.18	0.18	0.18	0.16	0.14	0.11	0.10	0.10	0.07	0.07	0.02	0.00	0.00	0.00
3	SHE	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	GOL	0.00	0.18	0.18	0.18	0.18	0.16	0.14	0.11	0.10	0.10	0.07	0.07	0.02	0.00	0.00	0.00
5	TMC	0.13	2.34	2.34	2.34	2.34	2.69	2.34	**2.00**	1.76	1.76	1.42	1.42	0.71	0.35	0.35	0.35
6	QUM	0.26	2.63	2.63	2.63	2.63	3.10	3.57	3.12	2.80	2.80	2.35	2.35	1.41	0.93	0.93	0.93
7	SC	0.39	2.38	2.38	2.38	2.38	2.88	3.39	3.88	3.53	3.53	3.04	3.04	2.02	1.50	1.50	1.50
8	CG	0.48	4.39	4.39	4.39	4.39	5.49	6.58	7.63	8.38	8.38	7.30	7.30	5.10	3.99	3.99	3.99
9	MON	0.48	0.05	0.05	0.05	0.05	0.07	0.08	0.09	0.10	0.10	0.09	0.09	0.06	0.05	0.05	0.05
10	REV	0.60	1.51	1.51	1.51	1.51	2.01	2.51	2.99	3.33	3.33	3.82	3.82	2.81	2.30	2.30	2.30
11	TEC	0.60	0.12	0.12	0.12	0.12	0.16	0.20	0.24	0.27	0.27	0.31	0.31	0.23	0.19	0.19	0.19
12	COM	0.87	1.35	1.35	1.35	1.35	2.68	4.02	5.29	6.19	6.19	7.49	7.49	10.17	8.81	8.81	8.81
13	PM	1.00	0.00	0.00	0.00	0.00	1.18	2.36	3.48	4.28	4.28	5.44	5.44	7.80	9.00	9.00	9.00
14	PRE	1.00	0.00	0.00	0.00	0.00	1.40	2.80	4.14	5.09	5.09	6.46	6.46	9.28	10.70	10.70	10.70
15	JC	1.00	0.00	0.00	0.00	0.00	1.48	2.96	4.37	5.38	5.38	6.83	6.83	9.80	11.30	11.30	11.30
Column Sum			15.15	15.15	15.15	15.15	23.47	31.09	37.46	41.31	41.31	44.68	44.68	49.43	49.13	49.13	49.13

Table 5.7: The matrix of Weighted Attractiveness values and the Weighted Attractiveness Scores. The bottom row, labelled Column Sum, is what is shown in column D of Table 5.6. Actor 12 (COM) is underlined, showing the highest Weighted Attractiveness Score. The Weighted Attractiveness values are calculated using the Attractiveness values in Table 5.5 multiplied by the actor's Exercised Power with the following expression $e_i \times A_i\ (\Theta j)$.
Source: KAPSARC

5.4.4 Model 4: Power and Risk Attitude Weighted Proportional Voting

Model 4 differs from Model 3 in only one respect:

– The Attractiveness function can be curved, not necessarily linear. (Thus, Models 2 and 3 are both special cases of Model 4)

The fundamental purpose of curvature is to model an actor's willingness to compromise and to tolerate risk.

The simplest formulae for nonlinear Attractiveness are quadratic functions of distance. There are three special cases, which we have labeled with the curvature number R:

– If $R = +1$, then $A_i(\Theta) = 1 - [d_i(\Theta)]^2$

– If $R = 0$, then $A_i(\Theta) = 1 - d_i(\Theta)$

– If $R = -1$, then $A_i(\Theta) = [1 - d_i(\Theta)]^2$

Each case is graphed in Figure 5.3, where for concreteness we have used $\Theta_i = 0.25$. Note that when $R = +1$, the solid blue attractiveness curve falls off very little close to the peak, indicating that an actor with $R = +1$ would see little reason to resist

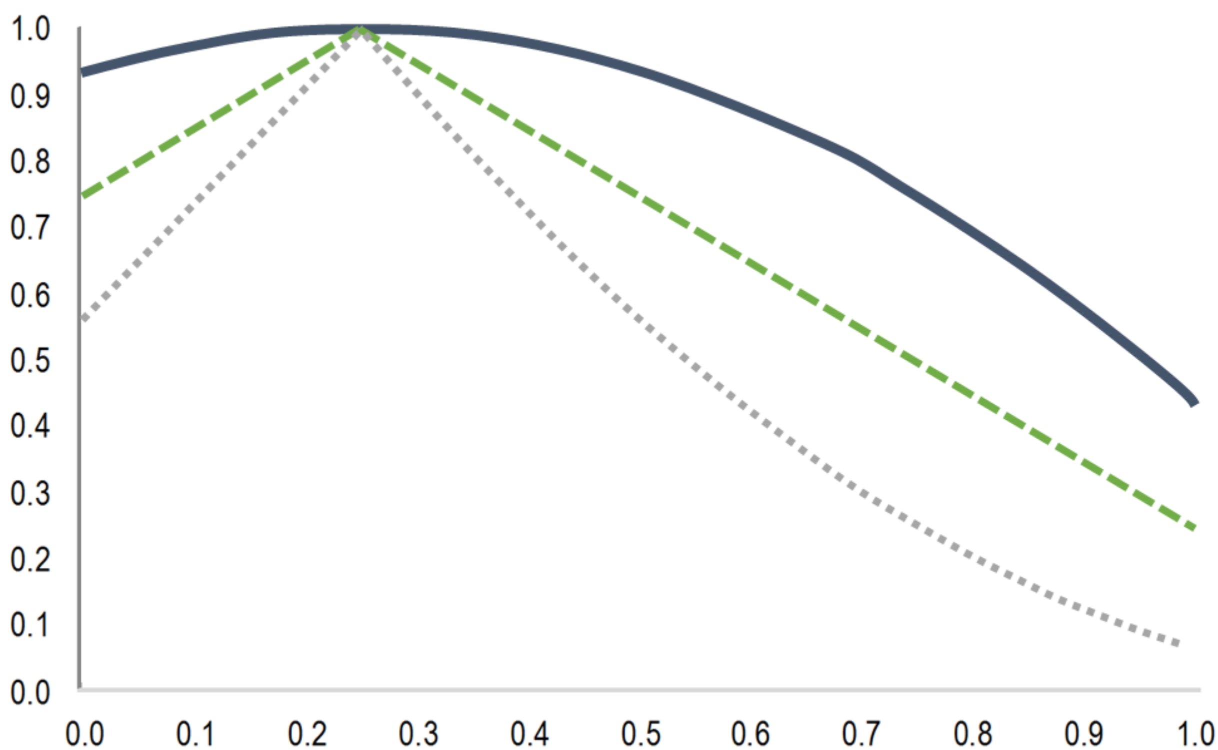

Figure 5.3: The shape of the Attractiveness curve is determined by the curvature number *R*. X-axis is Θ, the alternative Position. Y-axis is $A_i(\Theta)$, the Attractiveness to the actor of the alternative Position. The actor's Position is set to 0.25. R can vary between +/-1. When R = +1, risk aversion is modeled. One can see from the shape of the curve that the actor considers alternative Positions that are close to his advocated Position as highly Attractive. He is therefore likely to be willing to compromise, at least when the distance $d_i(\Theta)$ is small. When R = -1, the actor sees the Attractiveness of other Positions fall away quickly, even when the distance is very small. An actor with R = -1 is therefore unlikely to compromise and may be seen as risk-accepting. In real life, humans are rarely risk seeking. Even people who appear risk-seekers are most likely risk tolerent. It is a habit of humans to accept greater risk only when it brings the possibility of greater reward. A true risk-seeker would accept diminished returns if they offered more risk.
Source: KAPSARC

compromise. Conversely, when R = -1, the dotted grey attractiveness curve falls off very sharply near the peak, indicating that an actor with R = -1 would find even small compromises unattractive. The intermediate case R = 0 is indicated by the straight dashed green line. The possible values of R can also be interpreted in non-deterministic situations by their effect on the risk-aversion of actors. In non-deterministic situations, R = +1 is the most risk-avoiding, R = 0 is risk neutral, while R = -1 is the most risk-seeking.

The parameter R can be used to combine the three separate formulae into one as follows. The distance $d_i(\Theta)$ between the Position Θ_i of the i-th actor and some alterative Position Θ is the same as before:

$$d_i(\theta) = |\theta_i - \theta|$$

The Attractiveness function now consists of two terms. The first is the same as before, while the second incorporates the effect of the curvature R_i.

$$A_i(\theta) = (1 - d_i(\theta))(1 + R_i d_i(\theta))$$

For example, if $R_i = +1$,

$$A_i(\theta) = (1 - d_i(\theta))(1 + R_i d_i(\theta))$$

$$= 1 - d_i(\theta) + d_i(\theta) - [d_i(\theta)]^2$$

$$= 1 - [d_i(\theta)]^2$$

Similar derivations can be done for the other special cases $R_i = 0$ and $R_i = -1$. However, the advantage of using the curvature R is that it can be any decimal number between -1 and +1; in this way it can represent a continuous range of attitudes.

Model 3 used power-weighted voting to get the maximum attractiveness, based on a linear attractiveness function, which is equivalent to using R = 0 for every actor. This led to an outcome of 0.87; Bueno de Mesquita (1984a) forecast 1.00 for the outcome.

Based on the analysis in Scholz et al. (2011), we expect that the model most likely used in Bueno de Mesquita (1984a) is a version of power-weighted proportional voting, taking into account different risk attitudes of different actors. (Bueno de Mesquita's attractiveness function appears to be exponential, not quadratic as we have used above.) For this reason, we use the maximum weighted attractiveness score as the predictor. Because $R > 0$ corresponds to actors who are willing to compromise with others, and $R < 0$ corresponds to actors who are not willing to compromise, we might expect that the forecast would be influenced more by uncompromising actors, simply because the compromising actors would be willing to accept proposals favored by the uncompromising actors, and not vice versa.

Therefore, we would expect that when the actors are assigned small negative values for R, the prediction should move closer to that in Bueno de Mesquita (1984a), and this is indeed what occurs. Table 5.8 gives the results when R = -0.13 for all actors. Any $R \leq -0.12$ gives the same forecast: actors PM, PRE, and JC at Position 1.0, which is the same result as Bueno de Mesquita (1984a).

The $A_i(\Theta_j)$ matrix for this example is given in Table 5.9. We will now illustrate how WAS is calculated by looking at the attractiveness to TMC, actor 5, of the Position advocated by SC, actor 7. The shaded row of Table 5.9 is the row corresponding to (actor) i=5; the shaded column of Table 5.9 is the column corresponding to (Postion) j=7.

The Position of TMC is Θ_5 = 0.13, the Position in the fifth row.

The Position of SC is Θ_7 = 0.39, the Position in the seventh column.

A	B	C	D	E
Order	Actor	Position	Weighted Attractiveness Score	Forecast Rule
ID	A	Θi	$\sum_{i=1}^{15} e_i A_i(\theta_j)$	Maximum WAS
1	TAB	0.00	14.34	
2	SHA			
3	SHE			
4	GOL			
5	TMC	0.13	22.22	
6	QUM	0.26	29.58	
7	SC	0.39	35.87	
8	CG	0.48	39.72	
9	MON			
10	REV	0.60	43.05	
11	TEC			
12	COM	0.87	48.29	
13	PM	1.00	48.32	✓
14	PRE			
15	JC			

Table 5.8: Results for Model 4. The actors, their Positions and Weighted Attractiveness Sum (WAS) are listed in columns B, C and D. In this model the Attractiveness curve is non-linear. The full matrix of Weighted Attractiveness Scores is shown in Table 5.10, which may be counted as the third in the series following Tables 5.4 and 5.9. Under this formulation, actors 13, 14 and 15 (PM, PRE and JC), share the Position (1.0) with the highest WAS. This is the same outcome as Bueno de Mesquita reported.
Source: KAPSARC

			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
			TAB	SHA	SHE	GOL	TMC	QUM	SC	CG	MON	REV	TEC	COM	PM	PRE	JC
			0.00	0.00	0.00	0.00	0.13	0.26	0.39	0.48	0.48	0.60	0.60	0.87	1.00	1.00	1.00
1	TAB	0.00	1.00	1.00	1.00	1.00	0.85	0.71	0.58	0.49	0.49	0.36	0.36	0.12	0.00	0.00	0.00
2	SHA	0.00	1.00	1.00	1.00	1.00	0.85	0.71	0.58	0.49	0.49	0.36	0.36	0.12	0.00	0.00	0.00
3	SHE	0.00	1.00	1.00	1.00	1.00	0.85	0.71	0.58	0.49	0.49	0.36	0.36	0.12	0.00	0.00	0.00
4	GOL	0.00	1.00	1.00	1.00	1.00	0.85	0.71	0.58	0.49	0.49	0.36	0.36	0.12	0.00	0.00	0.00
5	TMC	0.13	0.85	0.85	0.85	0.85	1.00	0.85	**0.72**	0.63	0.63	0.49	0.49	0.24	0.12	0.12	0.12
6	QUM	0.26	0.71	0.71	0.71	0.71	0.85	1.00	0.86	0.76	0.76	0.63	0.63	0.36	0.24	0.24	0.24
7	SC	0.39	0.58	0.58	0.58	0.58	0.72	0.86	1.00	0.90	0.90	0.76	0.76	0.49	0.36	0.36	0.36
8	CG	0.48	0.49	0.49	0.49	0.49	0.63	0.76	0.90	1.00	1.00	0.86	0.86	0.58	0.44	0.44	0.44
9	MON	0.48	0.49	0.49	0.49	0.49	0.63	0.76	0.90	1.00	1.00	0.86	0.86	0.58	0.44	0.44	0.44
10	REV	0.60	0.36	0.36	0.36	0.36	0.49	0.63	0.76	0.86	0.86	1.00	1.00	0.71	0.57	0.57	0.57
11	TEC	0.60	0.36	0.36	0.36	0.36	0.49	0.63	0.76	0.86	0.86	1.00	1.00	0.71	0.57	0.57	0.57
12	COM	0.87	0.12	0.12	0.12	0.12	0.24	0.36	0.49	0.58	0.58	0.71	0.71	1.00	0.85	0.85	0.85
13	PM	1.00	0.00	0.00	0.00	0.00	0.12	0.24	0.36	0.44	0.44	0.57	0.57	0.85	1.00	1.00	1.00
14	PRE	1.00	0.00	0.00	0.00	0.00	0.12	0.24	0.36	0.44	0.44	0.57	0.57	0.85	1.00	1.00	1.00
15	JC	1.00	0.00	0.00	0.00	0.00	0.12	0.24	0.36	0.44	0.44	0.57	0.57	0.85	1.00	1.00	1.00

Table 5.9: The intermediate matrix of unweighted attractiveness values for Model 4, incorporating the curvature parameter, R. Note that it builds on Table 5.4, but diverges from the answers in Table 5.5 because of the non-linear curvature.
Source: KAPSARC

The distance between them is $d5(\Theta_7) = | 0.13 - 0.39 | = 0.26$.

At this point the calculation diverges from Model 3, because we use the curvature parameter R_5= -0.13. The attractiveness to TMC of the Position held by SC is

$$A_5(\theta_7) = (1 - d_5(\theta_7)) \times (1 - 0.13\, d_5(\theta_7))$$

$$= (1 - 0.26) \times (1 - 0.033) = 0.72$$

This is the number in bold at the intersection of the fifth row and the seventh column of Table 5.9. This process can be repeated for every row and every column in order to fill out the Attractiveness matrix.

Given the new Attractiveness values, we calculate the weighted sum of Attractiveness exactly as in Model 3.

The exercised power of TMC is e_5 = 2.69, as per Table 5.2.

The weighted Attractiveness is 2.69 x 0.74 = 1.94; this is the bold number at the intersection of the fifth row and seventh column of Table 5.10.

This process can be repeated for every entry in the seventh column, giving all the highlighted numbers in the seventh column. The weighted Attractiveness sum for the seventh column is the sum of the attractiveness values on the seventh column, which is 35.87.

This process can be repeated for every column, giving the row of column sums, one for each Position. Because we have used a different model, with a different model of Attractiveness, we obtain a different forecast: Position 1.0 occupied by PM, PRE, and JC, which is underlined in Table 5.10.

			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
			TAB	SHA	SHE	GOL	TMC	QUM	SC	CG	MON	REV	TEC	COM	PM	PRE	JC
			0.00	*0.00*	*0.00*	*0.00*	*0.13*	*0.26*	*0.39*	*0.48*	*0.48*	*0.60*	*0.60*	*0.87*	*1.00*	*1.00*	*1.00*
1	TAB	*0.00*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	SHA	*0.00*	0.18	0.18	0.18	0.18	0.16	0.13	0.11	0.09	0.09	0.07	0.07	0.02	0.00	0.00	0.00
3	SHE	*0.00*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	GOL	*0.00*	0.18	0.18	0.18	0.18	0.16	0.13	0.11	0.09	0.09	0.07	0.07	0.02	0.00	0.00	0.00
5	TMC	*0.13*	2.30	2.30	2.30	2.30	2.69	2.30	**1.94**	1.69	1.69	1.33	1.33	0.64	0.31	0.31	0.31
6	QUM	*0.26*	2.54	2.54	2.54	2.54	3.05	3.57	3.07	2.72	2.72	2.24	2.24	1.30	0.84	0.84	0.84
7	SC	*0.39*	2.26	2.26	2.26	2.26	2.79	3.34	3.88	3.49	3.49	2.95	2.95	1.89	1.38	1.38	1.38
8	CG	*0.48*	4.12	4.12	4.12	4.12	5.24	6.40	7.54	8.38	8.38	7.18	7.18	4.84	3.72	3.72	3.72
9	MON	*0.48*	0.05	0.05	0.05	0.05	0.06	0.08	0.09	0.10	0.10	0.09	0.09	0.06	0.04	0.04	0.04
10	REV	*0.60*	1.39	1.39	1.39	1.39	1.89	2.40	2.90	3.27	3.27	3.82	3.82	2.72	2.19	2.19	2.19
11	TEC	*0.60*	0.11	0.11	0.11	0.11	0.15	0.19	0.24	0.27	0.27	0.31	0.31	0.22	0.18	0.18	0.18
12	COM	*0.87*	1.20	1.20	1.20	1.20	2.43	3.70	4.96	5.88	5.88	7.24	7.24	10.17	8.66	8.66	8.66
13	PM	*1.00*	0.00	0.00	0.00	0.00	1.05	2.13	3.21	3.99	3.99	5.16	5.16	7.67	9.00	9.00	9.00
14	PRE	*1.00*	0.00	0.00	0.00	0.00	1.24	2.53	3.81	4.75	4.75	6.13	6.13	9.12	10.70	10.70	10.70
15	JC	*1.00*	0.00	0.00	0.00	0.00	1.31	2.68	4.02	5.01	5.01	6.47	6.47	9.63	11.30	11.30	11.30
			14.34	14.34	14.34	14.34	22.22	29.58	35.87	39.72	39.72	43.05	43.05	48.29	48.32	48.32	48.32

Table 5.10: The matrix of Weighted Attractiveness values and the Weighted Attractiveness Scores. The bottom row, labelled Column Sum, is what is shown in column D of Table 5.8. Actors 13, 14 and 15 (PM, PRE and JC) are underlined, showing the highest Weighted Attractiveness Scores. The Weighted Attractiveness values in the matrix are calculated using the Attractiveness values in Table 5.9 (which have incorporated the curvature parameter) multiplied by the actor's Exercised Power with the following expression $e_i \times A_i(\Theta_j)$.
Source: KAPSARC

6. The KTAB framework

6.1 Section summary

The formal notation for describing voting models and domain-specific utility models is presented in order to derive the forecasting rules of section 5. The notation is illustrated by describing not only formal elections but also generalized exertion of power.

Section 4 outlined some basic theory. Section 5 presented four ways of modeling that theory. The models can all be built in KTAB using the same basic framework. This framework is designed not to predict the outcome of CDMPs, but to guide the analyst towards the range of plausible outcomes. This framework has a set of common components: a set of outcomes, a numerical spectrum representing the range of possible outcomes, a specific model of Attractiveness, and a voting rule which defines the exertion of influence.

Within this framework, the four models can be succinctly compared.

- Model 1 allowed only binary voting but let different actors have different weights.
- Models 2, 3, and 4 built on each other, getting incrementally more sophisticated and general. All allowed proportional voting (in contrast to Model 1), but the treatment of actor weights, position attractiveness, and actor risk attitudes all became progressively more flexible.

In this section we present the formal logic that describes the algebra behind various voting models. We compare different voting methods and give examples of their use. One-person, one vote systems give each actor a single vote to be cast for a single option. Approval voting systems allow actors to vote in favor of all the options of which they approve. Cumulative voting grants each actor a set number of votes which can be distributed across options in any way. The difference between these systems allows for very different outcomes. Their correct modeling and appropriate use determines the validity of model outputs. The formal algebraic notation is set forth.

Different voting rules are only one way of defining a generalized voting system. Attractiveness can also be used to determine how actors exert their influence towards various possible outcomes. Again, the formal algebra is set forth for three different voting rules and then – last – for the Central Position Theorem outlined in section 4.4.

6.2 Introducing the Framework

The four models presented are all instances of the kind of models which can be built using KTAB. It will be helpful for purposes of comparison to indicate the main elements of the KTAB framework. The framework is designed to produce estimates of the relative likelihood of different outcomes, over complex issues. Technically, it estimates probability distributions over multi-dimensional, discrete, or mixed issues. The key elements of our framework may be summarized as follows:

- **The set of actors, which is symbolized as $\mathcal{A}$.** In each of the four examples, $\mathcal{A}$ is the same set of fifteen actors identified in Table 5.1.
- **The representation of possible outcomes.** In the four example models, the representation is the set of values in the range from 0 to 1. In our example question, each value represents a degree of government control over the economy. The fundamental representation in KTAB is of multi-dimensional or discrete outcomes, of which the one -dimensional is just the most intuitive illustration.

- **Domain-specific utility model (DSUM).** This is the method to estimate how attractive each outcome would be to each actor. In these four example models, Attractiveness is peaked at the actor's advocated Position, and declines (linearly or nonlinearly) with increasing distance. All four examples have the same DSUM: a quadratic function of distance, parameterized by the center of the Position and by curvature. In general, the structure of the DSUM depends on the set of possible outcomes: the DSUM over a set of discrete choices (e.g. which provisions to incorporate into or exclude from a proposed law) will obviously be different than one over a one-dimensional spectrum or a multi dimensional vector.
- **Voting rule.** This is the method to estimate how much Influence an actor will exert to promote one outcome over another. The voting model uses the DSUM to estimate what the perceived stakes will be for each actor.

With a more detailed framework in place, we can now succinctly summarize the four models. Each is an instance of the one-dimensional spatial model of politics, varying just three components:

Model 1

- Voting rule: binary
- Weights: unequal
- Utility function: declining in distance, but otherwise unconstrained

Model 2

- Voting rule: proportional
- Weights: equal
- Utility function: declining in distance, linear

Model 3

- Voting rule: proportional
- Weights: unequal
- Utility function: declining in distance, linear

Model 4

- Voting rule: proportional
- Weights: unequal
- Utility function: declining in distance, quadratic

In each model, the forecasting rule is to estimate the final outcome of the simple dynamic negotiation process. For these four models, the deterministic forecasting rule singles out the Condorcet Winner; the exact form of the Condorcet Winner varies as the three components change.

As mentioned earlier, the voting model is a formula that represents how much Influence an actor will exert to promote one outcome over another. In symbols, $v_i(x{:}y)$ is the Influence which the actor i would exert to promote outcome X over outcome Y. It is very important that the intent is to model the generalized exertion of Influence by various formal or informal means.

The net Influence of the whole group to promote option X over option Y is the sum of the individual Influences exerted:

$$V(x:y) = \sum_i v_i(x{:}y)$$

As mentioned earlier, the Condorcet winner is that option X which the whole group prefers to any distinct Y, i.e. with

$$\forall y \neq x \;\; V(x{:}y) > 0$$

6.3 Matrix representation of formal voting systems

So far, we have presented voting rules only as a way of describing how actors compare one alternative to another in a simple dynamic negotiation process. However, traditional elections are one-shot collective choices between a fixed set of options. They ask actors to cast their ballot for a particular option, not to compare pairs of options. We will now show how traditional elections can be cast in the KTAB framework.

In a traditional one-person, one-vote system, each option's final score is the number of individual votes it receives. In a corporate election, each option's final score is the number of shares voted in its favor. In either system, the vote of each actor for each option can be arranged in a matrix, with one row for each voter and one column for each option. The matrix representation can describe a multitude of voting systems – each system is defined by how each actor is allowed to fill in their row. Just some of the possibilities are the following:

- One-person, one-vote: Each actor can have "1" in only one entry, with the rest zero
- Approval voting: Each actor marks "1" on every option of which they approve
- Cumulative voting: Each actor gets the same number of votes, which they can distribute over options as they see fit
- The two matrices (Tables 6.1 and 6.2) illustrate the point more concretely

In the matrix for the "one man, one vote" system, each actor can place one vote for one candidate. In this example, we have six actors and three options. Actor 1 casts a single vote for option C; actor 2 cast a single vote for option A; and so on. Summing down the columns, we can see that the option with the highest total vote is C, with a total of 3 votes (see Table 6.1).

In this system, the only way a voter can exert Influence is by casting one ballot for one candidate. If actor i casts one vote for candidate X, they cannot also vote for Y, so the difference is $v_i\ (x : y) = +1$. If actor i casts one vote for candidate Y and none for X, then the difference is $v_i\ (x : y) = -1$. If actor i does not vote for either X or Y, then $v_i\ (x : y) = 0$. Because each actor has just one vote, which they cannot split across candidates, this is an example of binary, yes-or-no voting.

Corporate shareholder meetings do not follow the "one person, one vote" rule. Actors have a voting

	Option A	Option B	Option C
Actor 1	0	0	1
Actor 2	1	0	0
Actor 3	1	0	0
Actor 4	0	0	1
Actor 5	0	0	1
Actor 6	0	1	0
Total	**2**	**1**	**3**

Table 6.1: The "one person, one vote" system of binary voting. The actors can each vote for one of the three available options. The option with the most votes wins.
Source: KAPSARC

	Option A	Option B	Option C
Actor 1	4	3	4
Actor 2	5	1	4
Actor 3	8	0	2
Actor 4	4	3	3
Actor 5	1	2	7
Actor 6	6	2	2
Total	**28**	**11**	**22**

Table 6.2: A cumulative voting system. Each actor has ten votes it can cast as desired across the three options. Again, the option with the most votes wins. This is a form of proportional voting.
Source: KAPSARC

weight proportional to the number of shares which they own and they are allowed to put their total vote on exactly one option. If we symbolize the voting weight of each actor i by w_i, the weighted binary voting rule is as follows:

– if actor i votes for X then $v_i\,(x : y) = +w_i$

– if actor i votes for Y then $v_i\,(x : y) = -w_i$

– if actor i votes for neither X nor Y then $v_i\,(x : y) = 0$

Weighted binary voting is the voting rule used in Model 1.

The same matrix representation can be used for the cumulative voting example. We again have six actors and three options, but now each actor has ten votes to distribute. Summing down the columns of Table 6.2, we can see that the option with the highest total vote is A, with a total of 28 votes.

We now introduce formal notation for the matrix representation of voting systems. The entry for actor i and option j is designated by the symbol $v_i(j)$; in the cumulative voting example, $v_3(A) = 8$. Under any of the voting systems, the total vote for a option j can be represented by the symbol $V(j)$. It is just the sum down that column of each actor's vote:

In this table, *V(A) =28, V(B)=11, and V(C)=22*

$$V(j) = \sum_i v_i(j)$$

The whole group favors candidate j over option k when the former gets a higher total vote: $V(j) > V(k)$. This is equivalent to a positive difference between their total scores:

$$V(j) - V(k) > 0$$

The candidate which the group favors over all others is obviously the one with the highest vote, so that the difference is positive when compared to any other candidate. We can denote the group preference between options j and k as follows:

$$V(j{:}\,k) = V(j) - V(k)$$
$$= \sum_i v_i(j) - \sum_i v_i(k)$$

With the data from this table, we see the following six comparisons:

$$V(A{:}\,B) = +17$$
$$V(A{:}\,C) = +6$$
$$V(B{:}\,A) = -17$$
$$V(B{:}\,C) = -11$$
$$V(C{:}\,A) = -6$$
$$V(C{:}\,B) = +11$$

As expected, the Condorcet winner in this case is the one with the highest score, option A.

In terms of the matrix, the following equation says that we add down the J-th column, add down the K-th column, then take the difference. But the order of adding and subtracting does not matter; we get exactly the same result if we take the differences between the J-th column and the K-th column, then add:

$$\sum_i v_i(j) - \sum_i v_i(k)$$
$$= \sum_i [v_i(j) - v_i(k)]$$

If we denote an individual's preference between two options by the difference in support they give:

$$v_i(j{:}\,k) = v_i(j) - v_i(k)$$

$$V(j{:}\,k) = \sum_i v_i(j{:}\,k)$$

Thus, any voting system in which actors assign scores to options can be cast in the KTAB framework. This includes systems like super-majority voting, top two options both win, and many more. Full details can be found in the KTAB Technical Documentation (forthcoming).

6.4 Utility-based representation of generalized voting

A matrix of actor-vs-option scores is just one way to define a generalized voting system. Another way is to determine votes from the Attractiveness of different options; indeed, the Attractiveness of different options is the underlying motivation for exerting Influence. We will describe three basic voting rules which are provided in KTAB by default. Again, we use $A_i(j)$ to denote the Attractiveness to actor i of option j. Each KTAB model has a DSUM to calculate $A_i(j)$.

6.4.1 Default voting rules

A. **The binary, yes-or-no voting rule** is simply that actors cast their votes in favor of their preferred option of the pair:

- if $A_i(j) > A_i(k)$ then $v_i(j : k) = +w_i$ In other words, if the Attractiveness of option j to actor i is greater than the Attractiveness of option k to Actor i, then Actor i will exercise his entire influence in favor of j over k.
- if $A_i(j) = A_i(k)$ then $v_i(j : k) = 0$
- if $A_i(j) < A_i(k)$ then $v_i(j : k) = -w_i$

In terms of the weighted binary voting, such as corporate boards, this just means that each actor casts their vote in favor of the option they find more attractive.

As mentioned earlier, the binary voting of Model 1 leads to a simple rule to determine the Condorcet Winner of the negotiation process: the weighted median Position, as derived and presented in Black (1948).

B. **The proportional voting rule** is simply that actors exercise Influence proportional to what is at stake for them between the two options:

$$v_i(j:k) = w_i(A_i(j) - A_i(k))$$

This is the voting rule used in Models 2, 3, and 4, albeit with different weights and DSUM. Model 2 uses proportional voting with equal weights, while Models 3, and 4 use proportional voting with unequal weights.

C. **The cubic voting rule**. A refinement of the proportional voting rule is that actors do not respond strongly to small gains or losses, but respond strongly to large gains or losses. One rule with this behavior is the cubic voting rule:

$$v_i(j:k) = w_i(A_i(j) - A_i(k))^3$$

One situation where the cubic voting rule applies is suggested by the lobbyist rule: “Focus benefits, diffuse costs.” The logic is that a million voters will not significantly resist a loss of one dollar each in taxes, while a favored corporation will strongly support a million-dollar tax benefit. Under the proportional voting rule, the support and opposition would precisely cancel, but under the cubic voting rule diffuse opposition is far less than the focused support.

Unlike proportional voting, the cubic voting rule does not generally lead to a unique Condorcet Winner. Therefore, KTAB’s fundamental representation of the CDMP is a stochastic Markov process that identifies a stable distribution over likely outcomes. When a Condorcet Winner does exist, it will be the most likely outcome. As mentioned earlier, the pre-built spatial model of politics in KTAB is actually non-deterministic, multi-dimensional, and highly parameterizable. KTAB provides “deterministic” forecasts of the outcome of the negotiating process simply by citing the most likely outcome. (Whether the distribution is sharply peaked around the forecast or nearly flat provides important information about the forecast’s reliability.) The deterministic forecasting rules described here are actually not implemented in KTAB, though they do precisely describe the behavior of the prebuilt model, given the appropriate parameterization.

6.4.2 The Central Position Theorem

The proportional voting rule also leads to a simple rule to determine the outcome of the negotiation process. The outcome is the Central Position, subject only to the assumption of proportional voting and a few technical requirements regarding closed sets and limits. In particular, no assumptions are required about the structure of the set of options or about the shape of the attractiveness functions. The options could be points on a one-dimensional PSPP, points in a multi-dimensional vector space, alternative organizational charts for constituting a new national government, and many more. The utility function could be many-peaked over one – or multi dimensional options, or discontinuous over discrete options. Attractiveness could be determined by simple distance between options on a one-dimensional PSPP, or it could be determined by a complex sub-model applied to complex and highly structured options.

Under proportional voting, the group preference between two options is the following:

$$V(j:k) = \sum_i v_i(j:k)$$

$$= \sum_i w_i\big(A_i(j) - A_i(k)\big)$$

$$= \sum_i w_i A_i(j) - w_i A_i(k)$$

Referring to the cumulative voting matrix, we can see that it does not matter whether we add the columns first then take difference, or whether we take differences then add them:

(4 + 5 + 8 + 4 + 1 + 6) -

(3 + 1 + 0 + 3 + 2 + 2)

= 17

or

(4 - 3) + (5 - 1) + (8 - 0) +

(4 - 3) + (1 - 2) + (6 - 2)

= 17

In symbols,

$$\sum_i w_i A_i(j) - w_i A_i(k)$$

$$= \sum_i w_i A_i(j) - \sum_i w_i A_i(k)$$

Because the two sums are equal, the group preference (which is equal to the first sum) is equal to the second sum:

$$V(j:k) = \sum_i w_i A_i(j) - \sum_i w_i A_i(k)$$

If we define the weighted attractiveness sum as follows:

$$\omega(j) = \sum_i w_i A_i(j)$$

We can see that

$$V(j:k) = \omega(j) - \omega(k)$$

Again, the CW is that option j with $V\ (j : k\ ) > 0$ for all other k, and this condition is met when j maximizes the Weighted Attractiveness Sum. In Wise (2010a, 2010b), this Position is termed the “central position”, and the above proof is given that it is always the Condorcet Winner; see Jesse (2011) page 26 and page 28, equations 4 through 6.

This is the forecasting rule used in Models 2, 3, and 4. In Model 2 the actors all have equal weight of exactly 1. In Models 3 and 4, the actors have unequal weights.

Under the binary voting rule, the shape of utility functions does not matter so long as they are single peaked. Under proportional voting, the shape does matter, and the shape of the utility function is the only difference between Models 2, 3, and 4. Models 2 and 3 have the same straight-line declining Attractiveness for every actor, while Model 4 allows different quadratic Attractiveness curves for each actor.

7. Post-solution analysis

7.1 Section summary

This section describes some forms of sensitivity analysis, successful uses of the spatial model to analyze real-world problems, and some limitations common to all formal analyses. The importance of using sensitivity analysis to address these limitations is emphasized.

Once a model has been built and the initial analysis run, the business of interpretation begins. Correctly interpreting model results so that their implications can be fully understood relies not only upon a good understanding of the real-world environment the model is trying to recreate, but also of the assumptions, reliability, and limitations of the model and results.

Post-solution analysis allows us to generate a spread of alternative results that can guide us to the reliability of the base case. Do small changes in inputs lead to large changes in outputs? Can we contrive to achieve a specified outcome with plausible changes to the input variables? What would worst case scenarios look like, and what effect do they have on the results? Of course, we can also look to see how different models give different answers and tease the implications of these differences. This is essentially what was done with the four models in section 5.

All models have limitations, those that KTAB allows us to build to investigate CDMPs included. Using an inappropriate voting rule will lead to unreliable results. Assuming equal weighted votes when the actors all have different weights will clearly undermine the legitimacy of the model outputs. But where appropriate assumptions have been made, KTAB style models have met with considerable analytical success. The model used by Bueno de Mesquita in his various papers has been tested on over a hundred years' worth of data (Bennet & Stam 2000a, 200b).

Not all types of questions can be analyzed with KTAB-style models; KTAB cannot replace econometric models. Data quality remains critical – out-of-date data will lead to spurious expectations. KTAB models provide plausible outcomes of CDMPs based on a snap-shot of actor profiles. If these snap-shots fail to take account of recent developments they may not accurately represent the actors, and so the outcomes will be misleading.

Where real-world outcomes depart from the model's simulations, we gain further information on the quality of our actor profiles and the robustness of our assumptions. As with all models, valuable insight can be obtained regardless of whether or not the simulation exactly matches the real world.

7.2 Interpreting the output

It is a commonplace among analysts that the real work begins after a model has been conceived, designed, implemented, tested, validated, and exercised to obtain a solution. In this post-solution analysis phase the analyst will test the results of the model to aid their interpretation and build a deeper level of understanding.

Correctly interpreting results is not only a function of conducting further tests and investigations. Full understanding of all the assumptions (both explicit and implicit) will also remind the analyst of the limitations of the models, regardless of the complexity of the post-solution analysis.

7.3 Techniques for post-solution analysis

Whichever model, or combination of models, is used in KTAB, we are interested in obtaining and reflecting upon not only our base case result but also a multiplicity of alternatives solutions. We call this the principle of solution pluralism. The various solutions of interest (SoIs) we can identify are instrumental in deepening understanding.

7.3.1 Sensitivity analysis

Sensitivity analysis examines the consequences of “small” changes to parameter values used in the base case.

We may choose to see what happens to the outcome of a CDMP if the most powerful actor held a different Position, or if actors with the winning Position held had different levels of Influence or Salience.

The variance-based sensitivity methods, described in Saltelli et al., (2000, 2004, 2008, 2010) and elsewhere, can be very useful.

7.3.2 Outcome reach analysis

With outcome reach, the analyst asks the question “What would it take?”. In this way we can search for conditions under which certain outcomes or types of outcomes may occur.

In Model 1, actor 12 (COM) won. A valid question could be: “What would it take for actors 10 and 11 (REV and TEC) to win?”. Note that this question, unlike sensitivity analysis, does not assume we look only at small changes to the model. Sensitivity analysis is about how small changes affect a model’s behavior. Outcome reach analysis asks how big the perturbations need to be to shift to a particular outcome.

A limitation of this style of analysis is that the analyst may require a large number of manually-directed queries to find good answers.

7.3.3 Robustness analysis

In robustness analysis we search for robust solutions. Given a particular solution to the model, the search for robust solutions may well lead us to new solutions. This is in distinction to sensitivity analysis in which, given a solution, we seek to assess its robustness by altering parameter values and observing the consequences for the performance of the given solution. Of course, in sensitivity analysis and indeed generally, a solution whose performance does not seriously degrade as parameter values are altered is said to be robust. In short, sensitivity analysis assesses the robustness of a particular solution, robustness analysis seeks (new and) robust solutions.

Robustness analysis comes, broadly speaking, in two forms:

- In robustness under uncertainty analysis, we specify worst case developments (e.g. changes in Θ_is or in e_is that are credible and that would adversely affect a favored Position), and we seek the best ameliorating responses to them.
- In robustness under risk analysis, we specify probability distributions on model components and then seek solutions that meet a probabilistic performance standard (e.g., with probability of at least x will yield a result of at least y).

Robustness analysis has been explored principally in the context of constrained optimization models (Kimbrough et al. 2011; Kuo 2014). It is also apt in our context of modeling CDMPs.

7.3.4 Model structure analysis

Model structure analysis is a fourth kind of post-solution analysis. It mainly considers alternative modules for a given model or alternative models entirely. In undertaking model structure analysis, each of the afore-discussed forms of post-solution analysis pertain.

Section 5.4 may be seen as a simple example of model structure analysis. Four different models were used, each giving a different answer. The different results are a product of the different assumptions. Model structure analysis helps us understand the effects of model assumptions.

7.3.5 Individual versus group sampling exploration

In individual exploration we systematically make changes to one variable at a time and observe the behavior of the model. In group sampling exploration we randomly change a number of variables at once and record the behavior of the model. We repeat this process a large number of times and observe the distribution or pattern of behavior of the model.

7.4 Understanding KTAB's limitations

Like all toolkits, KTAB is targeted at a certain class of applications. Applications which are in its target class are good candidates for KTAB models; applications outside the target class are not good candidates. Further, there are limitations which are common to all analytical approaches to all problems, and the models built with KTAB are also subject to them.

It is important to bear in mind that KTAB is not a single model, or even a small collection of models. It is a toolkit which developers can use to build a limitless variety of structurally different model designs. Each such design can be supplied with a limitless variety of numerical parameters. It is clear that not all parameter values will be accurate, and also that not all model designs will accurately reflect real-world dynamics. If one used evenly weighted proportional voting to analyze a corporate boardroom vote, one could not place much faith in the analysis because the real world situation is both structurally different (the actual situation has binary voting) and parametrically different (the actual situation has uneven weights, even to the point where one majority shareholder can overrule all other actors combined).

We will present three general areas where models in KTAB's target class have been empirically validated.

The model described in *Conflict Forecasting Project - Final Report* (Bueno de Mesuita, 1984b), and in *The War Trap* (Bueno de Mesuita, 1981), was validated on over a hundred years' of data on European wars during the course of model development. Though it uses a very similar abstract method to the particular one-dimensional spatial model of politics used in the Bueno de Mesuita (1984a) paper, the Conflict Forecasting Project model is not actually a spatial model; the representation of a Position is not by a single number but through a weighted graph. This suggests that there is validity to the abstract framework (of which KTAB is a further generalization) itself, as it has been successful in both spatial and non-spatial applications.

The Conflict Forecasting Project model was re-implemented as EUGene (Bennett & Stam 2000a, 2000b, EuGene) and independently re-verified on a different data set covering several hundred years of conflicts world-wide.

The particular one-dimensional spatial model of politics used in the Bueno de Mesuita (1984a) paper has been successfully used in thousands of studies over several decades. Not all these studies have been released to the public, but an overview is available in Feder (2002). While surprisingly high success rates (over 90%) are claimed, discussions with experts familiar with the field suggest that this is a result of carefully choosing problems for which the methodology was well-suited and rejecting problems for which it was ill-suited.

The one-dimensional spatial model of politics has also been used as the basis of statistical estimation of the implied parameters of legislative votes, covering the entire history of roll-call votes in the United States Senate (Poole & Daniels, 1985, Poole & Rosenthal, 1985, 1991, 1999, 2000, Poole 2005). The spatial model, and its implied negotiation process, was found to account for over 95% of the votes.

The generalized negotiation process described in Black (1948) has been successful in thousands of academic analyses over several decades, on topics ranging from the timing of union negotiations to the previously mentioned case of using the SMP to analyze roll-call votes in the United States Senate.

Applications which are not in its target class are not good candidates for modeling via KTAB. Some of them have already been mentioned in this paper, such as standard econometric models. Statistical regression to estimate consumer preferences also would not be a good candidate, though both kinds of models might be used in the DSUM of a KTAB model to help estimate the attractiveness of different options.

No model can be expected to produce high-quality results from low-quality data. If the input data is derived from interviews with experts, it is important that they be truly expert and up-to-date on the situation being analyzed. If the input data is derived from analysis of large data sets, then all the usual problems of representative sampling, statistical validity, hypothesis testing, and so on must be addressed to ensure that the data is sufficient for the purpose.

No model is complete enough to rule out the possibility of unanticipated events not represented in the model or anticipated by the model designers. For example, if KTAB is used to build a model of farm policy deliberations among the stakeholder groups in a particular country, but that country then suffers a massive invasion by a powerful neighbor, then it could reasonably be expected that considerations of national defense would trump all prior domestic issues. The "balance of Influence" would have been completely rearranged by the change in the Salience of competing issues (prior domestic differences suddenly lose Salience, national defense issues become very salient), the injection of powerful actors not even present in the original model, and so on.

Similarly, a KTAB model based on the assumptions of economic consequences to various actors could be overturned by an unexpected legal decision that dramatically changed the regulatory environment. The shifts in economic consequences of different policies would shift the actors' Attractiveness Values, which would again upset the balance of Influence in unexpected ways.

The balance of Influence is obviously dependent on the Influence and Salience of different actors. An unexpected victory by an actor previously considered to be weak, and without enough of a coalition to make up for that weakness, would probably lead many actors to revise upward their estimates of that actor's Influence. Conversely, an actor long considered quite powerful on the international stage could suddenly have a significant drop in their Salience for purely domestic political reasons, which would again upset the balance of Influence. This could then set off a period of rearrangement, shifting coalitions, and tests of strength until a new balance of Influence emerged.

Each of these examples suggest not only the vulnerability of any model (formal or informal) to "black swan" events, but also the importance of sensitivity analysis to uncover vulnerability to unmodeled, unexpected dynamics.

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Notes

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About the project

KAPSARC is developing the KAPSARC Toolkit for Behavioral Analysis (KTAB), an open source software platform, to support modeling and analysis of collective decision making processes (CDMPs). KTAB is intended to be the standard platform for analyzing bargaining problems, generalized voting models, and policy decision making. It is our intent to use KTAB to assemble the building blocks for a broad class of CDMPs. Typical models in KTAB will draw on the insights of subject matter experts regarding decision makers and influencers in a methodical, consistent manner; and then assist researchers to identify feasible outcomes that are the result of CDMPs.