

Toward the Integration of Policymaking Models and Economic Models

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Summary

The traditional economic approach to policy analysis is to utilize tools and methods developed within the field of economics and study the economic impact of one or more policies solely from an economic perspective. As a consequence, the “policies” are usually formulated and evaluated only by an assessment of the pure economic optimality of expected outcomes. Moreover, economic models typically treat policy choices as exogenously specified. Once policies are selected according to some exogenous process, then scenario analysis can be performed to simulate the economic impact of those policies.

On the other hand, explicit research into the policymaking process – and in particular models of the policymaking process, tend to oversimplify or ignore the economic outcomes of the policies debated among policymakers. In particular, modelers of what we broadly classify as collective decision making processes (CDMPs) – in other words, models that capture the process of political choice – tend to utilize simple, fixed models of economics, if any economic consideration is taken into account at all. The economic consequences, or the outcomes, of choices of actors are generally treated as the end points of models. The implications of policy choices, once arrived at, are typically left unexplored.

Either approach in isolation misses out on important interactions between political and economic phenomena. The economically optimal policy is rarely the option selected, and is often not even under consideration because of political considerations. Policies that may appear to be politically appealing at first blush can lose their charm once adopted, because of the economic

impact upon attempts to implement them. In this paper, we argue that politics and economics are inextricably intertwined, and can be modeled as such. We develop a hybrid quantitative model of economics and CDMPs that allows us to capture the process of policymaking based on utilities derived from an endogenous economic model.

We develop this model with the KAPSARC Toolkit for Behavioral Analysis (KTAB), which is an open source platform for building CDMP models. All KTAB models represent stochastic decision-making among comparatively small numbers of actors or stakeholder groups – more than one, but less than hundreds – within the paradigm of Probabilistic Condorcet Elections (PCE). Even in formal voting situations, such as in government or boardrooms, informal influence-based negotiation is truly what drives voting – an obvious example of this is lobbyists influencing U.S. senators – so we need a negotiation model that captures this behavior. The PCE is a general framework for modeling this type of informal policymaking process; this framework is extremely flexible, and lacks restrictive assumptions present in others.

In this paper, to demonstrate our approach, we develop a CDMP based on the PCE, in which actors represent sectors in a Leontief economy. Each actor negotiates and forms coalitions so as to maximize its (the actor’s) utility. The data we used are synthesized from real data representing nine sectors and three actors of production in the U.S. economy from 1981. Applied to these data, our model demonstrates that strategically sophisticated economic policies can be generated using an integrated CDMP framework.

Introduction

Government policy, even when explicitly focused on a country's economy, is often shaped by noneconomic considerations.

While it is quite possible to characterize and demonstrate which policies will produce the most economically efficient and beneficial outcome under a given set of conditions, and even to rank order policy alternatives according to economic criteria, the policymaker many times appears to be relying on some other decision-making process when selecting which policy to implement. Indeed, when an economic modeler makes a decision to conduct scenario analysis of a range of policy alternatives, the selection of policies is typically treated as some exogenous process – and usually just an arbitrary selection process.

Similarly, models of collective decision-making processes (CDMPs) typically rely on simple, fixed models of economics, if they use any economic model at all. Nevertheless, the point remains that models of CDMPs tend to grossly simplify, or to ignore, the economic outcomes of policy choices that are the results of such models, nor do they capture the potential for feedback loops from the economic consequences into the decision-making process. Note that we use the expression CDMPs to broadly cover models of decision-making processes, strategic models of negotiation, bargaining models, spatial voting models and other models that capture the process of political choice.

In reality, political choice and the economic consequences of politically selected policies are intimately interconnected, since each may affect the other. As such, this represents an opportunity to improve on the literature dedicated to quantitative modeling in both areas of research. More complex bottom-up models of economies and markets, especially in the energy arena, can endogenously incorporate both CDMPs and economics to produce research that is more relevant to the policymaker.

In this paper, we outline a novel approach to endogenizing some of the noneconomic interactions that shape the choice and implementation of economic policies. A fundamental understanding underlying our approach is that economic policies can have differential impacts across different regions, industries or social groups. Policymakers typically represent different constituent groups, whether formally or informally. Those constituents are often organized around regions, industry or social groups – i.e., those clusters of the populations facing the differential impact of policy. As such, we make the assumption that some level of popular support is needed for policymakers to remain in power, whether in a representative democracy or even in an autocratic state; hence policymakers will be responsive to their perceived constituents. This responsiveness will translate into willingness on the part of the policymaker to expend a certain amount of effort to design a policy which it (the policymaker) anticipates will benefit constituents, and thus ensure that it can remain in office.

In addition, explicitly noneconomic criteria such as religious, moral or ideological criteria often influence policy. Public choice economics assumes away all such disparate factors by assuming that only economic criteria matter, albeit economic criteria chosen by a few on behalf of the many. However, understanding how different policymakers weigh these disparate economic and noneconomic impacts can be critical to understanding the range of plausible policy choices.

We demonstrate here how a hybrid model that incorporates the CDMP with a reasonable economic model of utility can be used to account for this multiplicity of concerns in identifying policy choices that accommodate a broader range of concerns for policymakers. Our demonstrative model uses a general model of CDMPs called the Probabilistic Condorcet Election (PCE). In this example, each

actor in the PCE represents either a sector or factor of production. The actors' utilities for a policy specifying a complete revenue-neutral package of sector-specific taxes and subsidies are estimated using a simple Leontief input-output model. Note that we are not suggesting the Leontief input-output model is the best model of economic reality: it is simply used here to represent the economic effects that actors anticipate from proposed policies. The utility of a policy of taxation or subsidization is the anticipated share of gross domestic product for that sector after imposition of the tax. A subsidy can be thought of as a negative tax. Hence, the utility is complex and nonlinear, given the constant elasticity of substitution demand curves and the nature of interactions between sectors or factors in the Leontief model. In addition, because of the economic relationships between sectors, it is natural for their policymaking representatives to form coalitions to influence the decision-making process to align around a policy that is mutually beneficial to the sectors they represent. The data we use cover nine sectors and three factors

related to production for both domestic and export consumption. The data are synthesized from U.S. economic data published in the 1980s. Applied to these data, our model demonstrates that strategically sophisticated economic policies can be generated using CDMPs.

We have implemented this model using the KAPSARC Toolkit for Behavioral Analysis (KTAB), which is a freely available open source framework and toolkit for modeling and analysis of CDMPs. The source code for KTAB is freely available on Github: <http://kapsarc.github.io/KTAB/>. KTAB encodes various types and formulations of CDMPs, including the Spatial Model of Politics and PCE, in ways that are easily extensible. A good introduction to the capabilities of KTAB is available in Wise, Lester and Efirid (2015).

Graphically, our proposed framework for integrating the economic considerations with policy determination is shown in Figure 1.

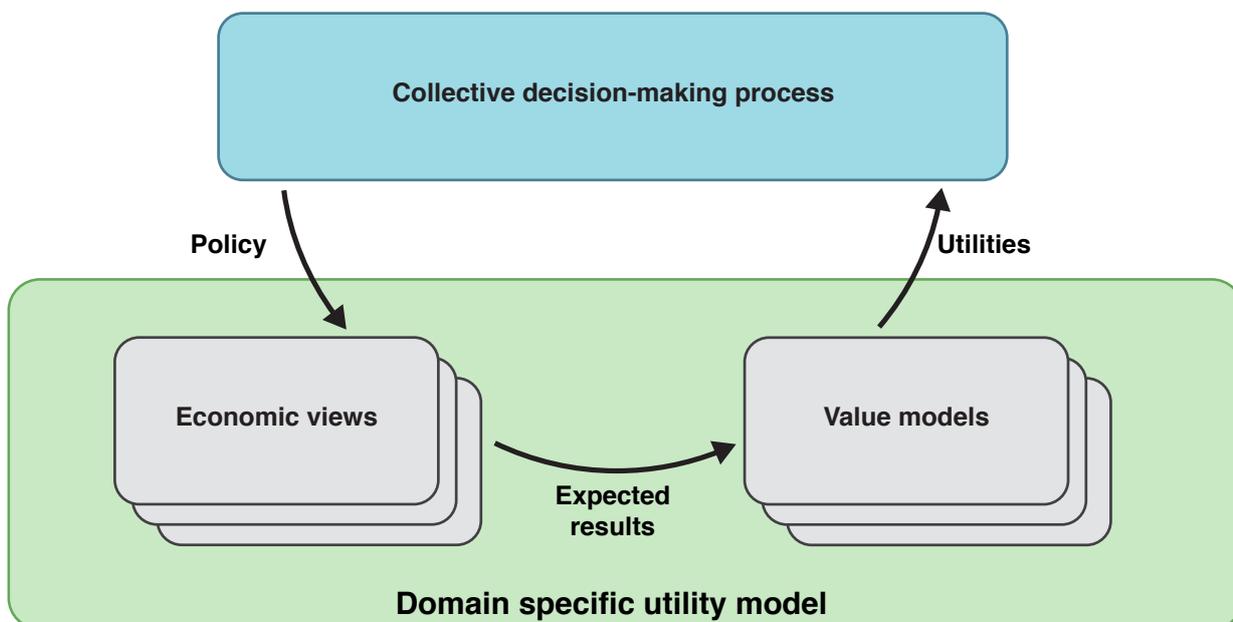


Figure 1. Integration framework.

Source: KAPSARC.

Introduction

This is also represented in the structure of the remaining paper. We continue with some basic background on quantitative CDMP models in Section 2, followed by a brief review of our implementation of a Leontief input-output model in Section 3. This is followed by our detailed

discussion of the data, their preparation and the results of our model in Section 4. We conclude the paper in Section 5 with some final remarks about the modeling framework and possible future directions for research.

CDMP Models

Policymaking is often conceived of as the collective decision-making of a group of actors, with each actor pursuing policies that serve a combination of its own interests along with the interests of its constituents. The class of CDMPs considered in this section differs from traditional economic models, which usually assume a very large, uncountable number of actors using a mechanism, such as a price determined through a market, to settle voluntary transactions in goods and services. A key characteristic of policymaking in a CDMP is the comparatively small number of actors. In a CDMP, there are a finite, countable number of actors, each of which can be identified, although these actors may measure their self-interest by the expected reactions of a large number of constituents, allies, opponents, etc. In this context, an actor need not be an individual person, but could also be a coherent group with common interests, so long as it could reasonably be assumed that these behave as a unitary actor, such as ‘subsistence farmers’ or ‘aerospace manufacturers’. Individual leaders are taken as representatives of the groups they lead. This is entirely analogous to the use of a ‘representative household’ or ‘representative consumer’ in computable general equilibrium models. Broadly speaking, we can consider two different types of actors, based on how their interest is defined:

Utility maximizing actors: Actors that always vote between alternatives based on which policy outcome they prefer most, regardless of who proposed them or why.

Office seeking politicians: Actors that propose alternatives based only on how likely it is that their proposal will be selected by the group of utility maximizing actors.

Another key characteristic is that there is usually no price mechanism to mediate interactions. Although

there are no prices, actors in a CDMP generally have different levels of influence. Examples include voting blocs of differing size, interest groups with differing amounts of funds to support public relations campaigns, shareholders with different numbers of shares during a corporate meeting, and so on. Actors can combine their influence to form coalitions, and a strong coalition can force an outcome over the objections of a weaker coalition. The classic example is an election where the candidate with the most votes wins, even though the smaller voting blocs supported, and continue to prefer, different candidates. Also, unlike a market, there is only one winning candidate rather than each participant separately getting the candidate of their personal choice.

To obtain analytically tractable solutions, most CDMP models focus on the details of negotiation and employ very simplified models of how actors derive utility from policies. A typical example of a simplified CDMP model is the classic ‘divide the dollar’ model of bargaining, in which the actors divide up a fixed resource, and the utility to each actor would be exactly the proportion it received. By giving up the requirement for analytical tractability, we can instead develop simulation models which employ more complex and domain specific utility functions and negotiation processes. To accommodate these more complex situations, we have generalized the usual negotiation frameworks so that they still cover the classic cases, while also becoming usable for decision processes with less artificial models of utility. This approach, which we have taken in the KTAB framework, more realistically models the decision-making process and allows the researcher to analyze a broader class of situations.

In this section, we will briefly discuss some of the fundamental concepts of quantitative modeling of collective decision-making processes, then move

on to more details about the specific model used in this application. After presenting fundamental concepts, the four CDMPs examined are the following:

- Spatial Model of Politics and the Median Voter Theorem.
- Central Position.
- Probabilistic Condorcet Election.
- Symmetric negotiation, using PCE.

For more complete information, the interested reader is directed to the resources in the bibliography. In the sections following, we will begin by discussing models of voting. However, the concept of voting models can be quickly generalized to the exertion of influence, where neither formal voting, nor any kind of voting as such, actually takes place. Rather, the models generalize to the concept of the exertion of influence, which is fundamental to the concept of a CDMP.

Fundamental concepts

In a basic negotiation model, there are A actors, each of which advocates some option $\theta_i (i=1\dots A)$, such that $\theta_i \in \Omega$; Ω is the exclusive and exhaustive set of possible options. The set of positions advocated by the A actors at a given time in the negotiation process is called the state S , where $S \in \Omega^A$. When we are discussing the state of a specific iteration of the CDMP, we can add a time index S_t .

In a choice between any pair of options x and y , actors can exert influence to try to get the group to adopt one or the other. The influence exerted by the i^{th} actor is written as $v_i(x:y)$ where positive

values favor x and negative values favor y . The simplest formal voting system is ‘one person, one vote’ (1P1V): each actor exerts influence by casting a single indivisible vote for $x (v=+1)$, for $y (v=-1)$, or for some other option ($v=0$). No matter how individual votes between options are defined, the total group preference for x over y is simply the sum of the votes:

$$V(x:y) = \sum_{i=1}^A v_i(x:y). \quad (1)$$

We can split this into the total support for x and for y by looking at the influence exerted by supporters of either alternative. The coalition supporting x over y is the set of actors exerting positive net influence:

$$c(x:y) = \{i \mid v_i(x:y) > 0\}, \quad (2)$$

and we can compute the strength of the coalition supporting x over y as

$$s(x:y) = \sum_{i \in c(x:y)} v_i(x:y). \quad (3)$$

For the opposite perspective of coalitions supporting y over x , symmetric definitions of $c(y:x)$ and $s(y:x)$ apply. Clearly, the strength for an option in a pair is simply the sum of the votes cast by the coalition supporting that option, so the total group preference $V(x:y)$ is the excess of votes favoring x over those favoring y . We say that the option backed by more votes (influence) dominates the other. Note that in this formalism, there is generally no idea of ‘voting for x ’ alone: there is only pairwise ‘voting for x over y .’ Formal voting systems, like one person, one vote (1P1V), weighted voting, approval voting and so on, are special cases.

Generalizing from the 1P1V special case, in which a vote between two options is always $+1$, -1 , or zero, the same concepts hold in cases in which actors exercise influence measured as real values. The next simplest formal voting scheme, weighted voting, is like 1P1V, but with different numbers of votes assigned to each actor. A common example is weighting by shares during corporate board meetings. There are many ways to compute these real valued generalized votes, and we will come back to this. The simplest is binary voting, in which an actor exerts all its influence to support a preferred option, no matter how small the difference. This is shown here, where the influence of the i^{th} actor is w_i .

$$v_i(x:y) = \begin{cases} +w_i & u_i(x) > u_i(y) \\ -w_i & u_i(x) < u_i(y) \end{cases} \quad (4)$$

The same measure of group preference as in (1) can thus be represented as the net vote by comparing the real valued strength of supporters of each option:

$$V(x:y) = s(x:y) - s(y:x). \quad (5)$$

The situation where $V(x:y) > 0$ can also be written as $x \succ y$ to emphasize the static fact that x is chosen over y , or as when we wish to emphasize that x could follow y in a dynamic, step wise CDMP. An expression like $a \rightarrow b \rightarrow c \rightarrow d$ is a four-element dominance sequence, where each option is dominated by the following option. If there is one option which dominates all others, then we determine the winning option, x , by computing the net vote for all pairs of options and identifying the option, x , for which the net vote against every other option, y , is positive:

$$\forall y, x: V(x:y) > 0. \quad (6)$$

Since at least Condorcet (1785), dominance sequences such as this have been known to cycle indefinitely (i.e., $a \rightarrow b \rightarrow c \rightarrow d \rightarrow a \rightarrow \dots$) and have no winning option unless special conditions are imposed. If an option exists that dominates all others in pairwise contests, it is called the Condorcet Winner. The entire process of determining $v_i(x:y)$, $s(x:y)$, and $V(x:y)$ for a particular set of options (e.g., a state) is sometimes termed a Condorcet election over those set options.

An option, x , is the Condorcet Winner if, and only if, in every pairwise contest the coalition favoring x is stronger than that favoring the alternative. The critical factor is not the strength of whichever actor (if any) most prefers x : the critical factor is the strength of the entire coalition advocating x when compared with some other position. A position, x , advocated by a comparatively weak actor with many strong allies could easily prevail against a position, y , supported by a comparatively strong but isolated actor. The membership and strength of a coalition is affected by both intrinsic capability and the perceived stakes for each actor, which in turn are combined by the voting rule to estimate exercised influence. We will return to the voting rule shortly.

When the winner of a Condorcet Election is determined strictly by (6), regardless of how small the margin of victory is, we refer to the winning option as the Deterministic Condorcet Winner. In many situations, this is too simplistic. When we model informal voting as the exertion of informal influence, the outcome is better represented as a probabilistic function of the ratio of coalition strengths. We will discuss this in a later section.

Spatial Model of Politics and the Median Voter Theorem

Due to cyclic group preferences, a dynamic CDMP is not guaranteed always to have a Condorcet

Winner; this requires the imposition of special conditions. One such framework that has been shown to guarantee existence of a Condorcet Winner is the Spatial Model of Politics (SMP); the SMP was introduced by Black (1948), along with his Median Voter Theorem. The SMP ensures that a CDMP will arrive at a Deterministic Condorcet Winner by imposing some basic conditions:

- The set of options Ω can be arranged on a line segment.
- Each utility maximizing actor has a single peaked utility function centered on its most preferred option.
- Proposals are made by office seeking politicians and voted upon by utility maximizers.
- The amount of influence a utility maximizing actor brings to bear through binary voting is a function of capability and stakes, as per (4).
- Preponderance of influence determines the winning option in a vote, as per (5).

While not every policy debate can be usefully modeled as a gradation between two extremes, a great many can be modeled in this way. According to the experience cited in Feder (2002), thousands of political, military, and economic situations have been successfully analyzed using the Spatial Model of Politics. An independent application of the one-dimensional Spatial Model of Politics was used for detailed analyses of the entire voting history of the United States Congress; according to Poole and Rosenthal (1985; 1991; 1999; 2000), the Spatial Model of Politics was found to explain over 90 percent of congressional voting behavior.

Black's Median Voter Theorem is one of the most famous and widely used results in voting theory and

analytical politics. It is widely used in both scholarly articles and popular political discussions. In this theorem, he developed the idea of the weighted median position which dominates all other options. He demonstrated, pursuant to some conditions, two crucial facts. First, there is a Deterministic Condorcet Winner which is the weighted median position. Second, a dynamic CDMP based on the SMP would converge to that Deterministic Condorcet Winner. It is critical for the weighted median position that an actor's vote between two options be an increasing function of the distance between options x and y , and that actors prefer options closer to their own. Mathematically, this means that the utility to an actor is single peaked at its position, and monotonically decreasing elsewhere.

Black describes an iterative process where there is a proposal, z_t , 'on the floor' at each step, and actors propose amendments to it. The proposers are assumed to be office seeking politicians, who seek to have their proposal adopted by utility maximizing actors. As explained, they will be motivated to propose amendments that dominate the current proposal and the dominant amendment will be selected. Mathematically, we can represent the iterative convergence of the CDMP toward the Deterministic Condorcet Winner, where x is the amended proposal z_t , as

$$z_{t+1} = \arg \max_{x \in \eta(z_t)} V(x : z_t), \quad (7)$$

where η indicates a 'neighborhood' of proposal similarity. Because only dominating proposals are ever accepted under the Median Voter Theorem, z_t monotonically converges to the Deterministic Condorcet Winner, then the iterative process stops.

Unfortunately, the Median Voter Theorem does not generalize easily. The most relevant reason here is the requirement for a single peaked, monotonically decreasing utility function in one dimension. In our example, we have multiple tax rates on different sectors, so the actors' positions cannot be arrayed on a single dimension. The actors' utility functions are not single peaked because of the distinction between policy and consequences. The spatial model treats policies as points on a line and assumes that the difference in utility between policies depends only on the distance between the policies. However, in the case of taxes, it is possible to generate multiple tax vectors that have the same net effect on a particular actor. Thus, utilities to that actor are the same, even though the policies themselves might be quite different. Indeed, a policy 'further away' could be preferred to a nearby alternative. Hence, while the SMP is an important framework and a fundamental concept which can be generalized for more complex CDMPs, it is too constrained and simplistic for many applications.

Generalized voting and voting rules

We began Section 2.1 with the concept of formal voting. While this is appropriate for many formal voting processes, it would not be appropriate for modeling negotiation situations based on informal exertion of influence. Arguably, this is more fundamental than formal voting, as votes cast during formal voting are often a result of the prior informal negotiation. Generalized voting gives us the flexibility to model CDMPs that are much more realistic. Indeed, the system of generalized pairwise voting can be easily extended to any system where actors assign scores to options, simply by taking the difference between the scores. These and other broader extensions are described in Wise, Lester and Efirid (2015).

There are several plausible alternatives to binary voting, including proportional, cubic, and so on. Hybrid generalized voting rules can also be formulated, e.g., binary proportional or cubic proportional. Although the cubic and hybrid voting rules have some empirical plausibility, they do not have the special conditions required to yield a Deterministic Condorcet Winner in any simple cases. Proportional voting, however, is not only intuitively plausible, it also has very desirable mathematical properties and can yield a Deterministic Condorcet Winner.

Proportional voting is the exertion of the actor's influence in proportion to its perception of the stakes, meaning how much utility it stands to gain or lose from the two alternatives. The proportional voting rule, where $u_i(x)$ is the utility to actor i of option x , is as follows:

$$v_i = w_i [u_i(x) - u_i(y)]. \quad (8)$$

It is common to normalize utilities to the [0,1] range, so that the difference in utility between two options is always in [-1, 1], which keeps an actor's influence in the $[-w_i, +w_i]$ range. For example, if an actor can exert influence by casting 10 votes, $w_i=10$, his vote can range from +10 votes (for x) to -10 votes (for y).

Proportional voting is designed to model a wide choice of means of exerting influence, an implicit budget on exerting influence, an intrinsic cost to exerting influence, or all three. Typical examples include spending money to fund political relations campaigns, nations deploying military force to pressure other nations, special interest groups lobbying for tax and subsidy policies, and so on. Political parties and special interest groups expend their campaign funds carefully, expending little or no resources on minor issues and making the

largest expenditures on the most important issues, so binary voting is not an accurate model of their behavior. Proportional voting can be shown always to lead to a Condorcet Winner.

The Central Position

The Central Position Theorem was derived in Wise (2010) and published in Jesse (2011). A closely related but distinct case is analyzed in Corollary 4.4 of Coughlin (1992). The Central Position Theorem states that under proportional voting, the CDMP will lead to an outcome called the Central Position, which maximizes a Weighted Attractiveness Score; the whole group's balance of influence will favor this outcome over any other. In other words, it is the Deterministic Condorcet Winner under proportional voting. By almost the same logic as with the Median Voter Theorem, office seeking politicians will always have an incentive to propose options with a higher Weighted Attractiveness Score until the Central Position is proposed. At that point, no other option can displace the Central Position and no further negotiation will occur because every actor knows that further proposals will be rejected.

In the one-dimensional spatial model with binary voting, the detailed shape of an actor's attractiveness curve is irrelevant: actors exert their full voting weight in favor of whichever alternative is closer to their own position. With proportional voting, the shape of the attractiveness curve is necessary to compare the attractiveness of two options and gauge how much influence an actor will exert for one or the other. The Central Position Theorem is an improvement in generality over the Median Voter Theorem for two main reasons.

First, the Central Position Theorem requires fewer restrictive assumptions and constraints to ensure

the existence of an outcome. The Median Voter Theorem states that the one-dimensional spatial model with binary voting gives the weighted median position as the Deterministic Condorcet Winner. In contrast, the Central Position Theorem states that proportional voting alone gives the Central Position as the Deterministic Condorcet Winner, with no further assumptions about the shape of the utility function or the set of options. There are some technical assumptions needed for both the Median Voter Theorem and the Central Position Theorem to ensure that the necessary limits exist, but they are generally not problematic. Maximizing the Weighted Attractiveness Score is transitive and acyclic under proportional voting, as will be shown below. This means that all the problems related to circular preferences and unstable outcomes are eliminated.

Second, the Central Position Theorem can easily be applied to economic CDMPs with multiple numeric parameters, rather than just the one-dimensional spatial model. Indeed, the logic of the Central Position Theorem is fundamentally non-spatial, which makes it very broadly useful, but also harder to visualize. The strong assumptions that made the Median Voter Theorem easy to visualize are no longer needed, so there is no simple geometric explanation of the Central Position Theorem. This is intimately connected to the fact that the Central Position Theorem applies not only to spatially arrayed problems, but also to discrete combinatorial problems in which notions of distance are not useful.

To arrive at the Central Position in a CDMP, we begin with the same formulation of group preference between a pair of options, x and y , based on the influence exerted by interest groups. Note that we compare influence (e.g., number of ballots), not interpersonal utilities.

Under proportional voting, this becomes:

$$\begin{aligned} V(x:y) &= \sum_{i=1}^A v_i(x:y) \\ &= \sum_{i=1}^A w_i [u_i(x) - u_i(y)]. \end{aligned}$$

Factoring the right hand side, we get:

$$V(x:y) = \sum_{i=1}^A w_i u_i(x) - \sum_{i=1}^A w_i u_i(y).$$

From this, we can see that the Central Position is the option x which maximizes the following Weighted Attractiveness Score:

$$\zeta(x) = \sum_{i=1}^A w_i u_i(x). \quad (9)$$

Notice that $x \succ y$ exactly when $\zeta(x) > \zeta(y)$. Because ζ is an ordinary number, $\zeta(x) > \zeta(y) > \zeta(x)$ is impossible, so cycles like $x y x$ are impossible. Note that this Weighted Attractiveness Score is not simply postulated as a social utility function to be maximized, it is derived from an underlying model of the negotiation process. This is entirely analogous to the total surplus that is maximized in classical free markets: maximization of total surplus is a logical consequence of a particular model of dynamic interactions, not an assumption. Again, note that the Weighted Attractiveness Score sums influence (e.g., votes) and does not make interpersonal comparisons of utility. In particular, for the two special cases of one person, one vote (1P1V) and weighted votes, ζ is just the total votes for the option, and the winner is the option with the most votes.

This Weighted Attractiveness Score has been independently rediscovered several times, under various specialized conditions. The earliest of

these appears to have been by Arrow and Maskin (1951), who termed it the ‘Benthamite Social Utility Function’. Each such rediscovery relied on a fairly detailed characterization of both the set of options and the utility functions, unlike the above proof, which may be the simplest and most general published to date. For example, this proof does not even require that the choices x, y , etc. be described by numerical values. They could be choices over alternative organizational charts, alternative matchings of political parties to ministerial portfolios, and so on. In these more general cases, it may be difficult even to define a distance function between options, i.e., a symmetric and non-negative difference which obeys the triangle inequality. Without a distance function, it is difficult to apply concepts like ‘unimodal,’ ‘convex,’ or ‘concave,’ so consequently many economic rationales cannot be applied. However, the above Weighted Attractiveness Score exists and the Central Position Theorem can be applied. The Central Position is thus defined to be the position which maximizes the Weighted Attractiveness Score under the proportional voting model of (8) and (9). Several such examples are provided in the KTAB software distribution on GitHub (available for download at <http://kapsarc.github.io/KTAB/>).

An interesting characteristic of the Weighted Attractiveness Score is that it can be multimodal over a vector space, depending on the characteristics of the actors’ utility functions. Because of this, the Central Position can jump discontinuously in response to small changes in utilities or capabilities, which is not possible for the weighted median position. While there are many analyses of why political parties do not always coalesce to the weighted median position in the one-dimensional case, and why political ‘tipping points’ can lead to discontinuous change, the Central Position Theorem provides a particularly succinct explanation of both phenomena.

As this point is relevant to our economic negotiation example, we explain how it arises in a simple one-dimensional case. We consider six actors, the positions of which are characterized by a number in the unit interval, so $\theta_i \in \Omega = [0, 1]$. The positions might correspond to something like a left-right political spectrum, level of government debt, or some other one-dimensional policy issue. Table 1 gives the positions of the actors, as well as the w_i values for maximum possible influence.

The behavior of the Central Position depends crucially on the shape of the actors' utility functions. While utility functions of goods are required to be increasing and convex, the utility functions here are not in terms of goods but in terms of political policies. Hence, there is no reason to expect them to be either concave or convex. We compare the cases of convex and concave variants of a quadratic utility.

If the utility functions of these actors are concave and unimodal at the positions of Table 1, then the Weighted Attractiveness Score has the jagged, two peaked shape in Figure 2(a). Any small changes in the curve can cause the Central Position to jump discontinuously between 0.84 and 0.25. In this case, the Central Position is always a position currently

held by one of the actors. Due to the concavity, the group will select one or the other, without compromise. Given that not all actors will have precisely the same estimates of policy parameters, they will have slightly differing estimates of the height of each peak. If there is uncertainty as to which of the two peaks is the real Central Position, then the office seeking politicians will separate into two distinct clusters, one around each peak.

On the other hand, if the utility functions are convex and unimodal at the positions of Table 1, then the Weighted Attractiveness Score forms the smooth, single-peaked curve in Figure 2(b). The Central Position happens to be at 0.54 for these data, a compromise position not currently held by any of the actors. Due to the convexity, the group will compromise and 'split the difference,' albeit weighted by relative strength. In addition, variations in the data cause the Central Position to shift smoothly as the single peak moves, rather than jumping discontinuously as in the prior case. Again, because actors will have different estimates of political parameters, they will choose different positions of slightly differing height. In this case, the band of uncertainty will cover a contiguous range around the Central Position, so the office seeking politicians will form one cluster.

Table 1. Positions and influences for six actors.

Actor	Position	Influence
1	0.13	23
2	0.19	15
3	0.25	74
4	0.84	71
5	0.89	27
6	0.93	13

Source: KAPSARC.

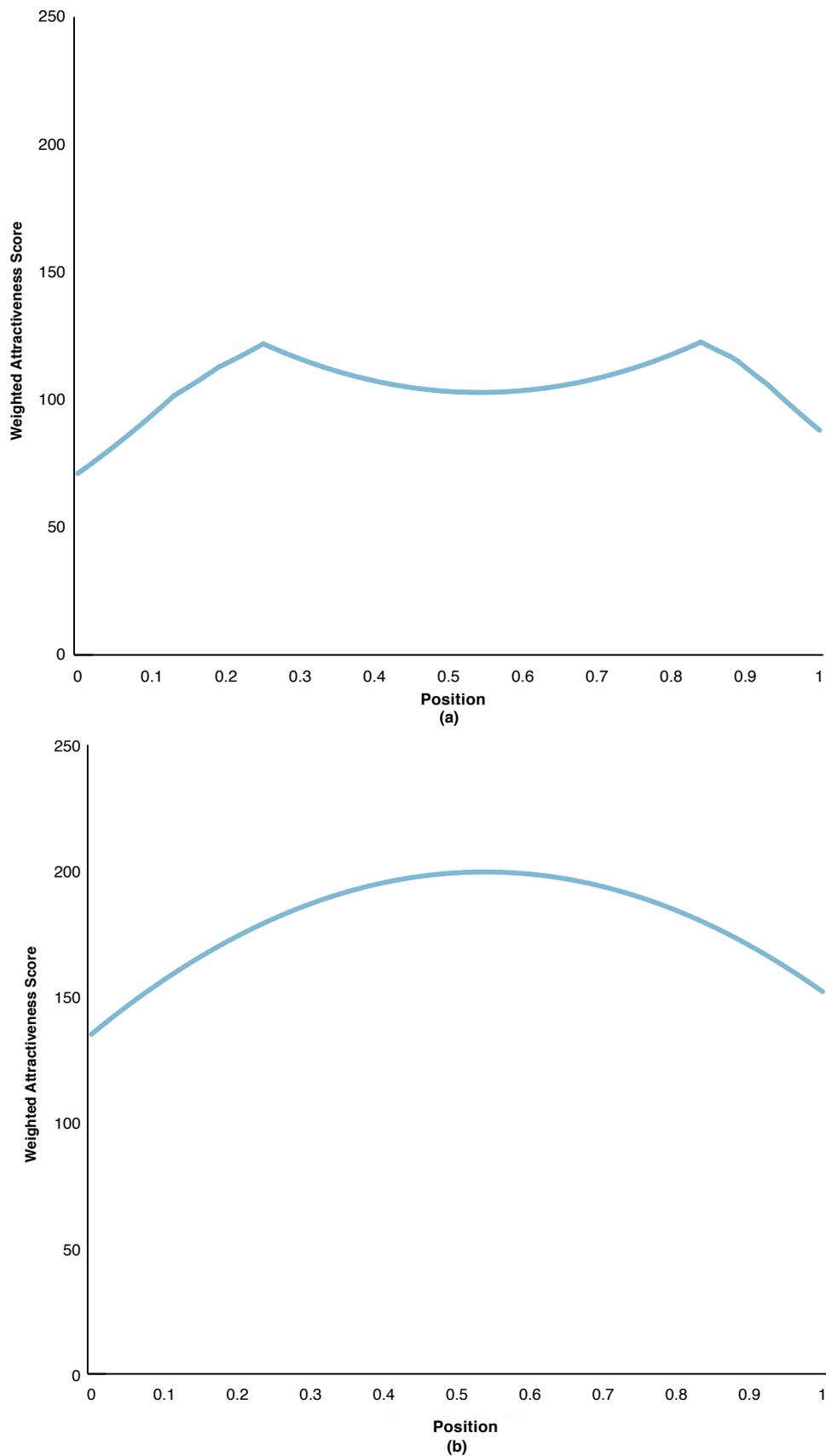


Figure 2. Weighted Attractiveness Score. (a) Pane: Concave Utilities; (b) Pane: Convex Utilities.

Source: KAPSARC.

Probabilistic Condorcet Election

By using (6), a Condorcet Election can result in a Deterministic Condorcet Winner that is guaranteed to win, even by a very small margin. It may be appropriate for a committee vote of 11:10 resulting in the first option always being a clear winner. However, it is inappropriate for modeling informal negotiation situations that are based on influence, rather than votes. If two rival political campaigns each spent \$10 million to support their candidate, then each candidate would have a 50 percent chance of victory, all else being equal. If they could fund campaigns in the ratio 11:10, the first candidate would not be absolutely guaranteed of victory, though the probability would be raised somewhat above 50 percent – again, all else being equal. More general informal negotiation processes such as this are better modeled with a Probabilistic Condorcet Election, in which the probability of an option’s winning is proportional to the ratio of coalition strengths. For a pair of options, x and y , one rule for the probability that x will be chosen over y is that the probabilities depend on a simple ratio of strengths, as in the following:

$$P[x \succ y] = \frac{s(x:y)}{s(x:y) + s(y:x)}. \tag{10}$$

For the example of the rival political campaigns funded at the 11:10 ratio, this would result in a 52 percent probability of success for the first campaign. We can think of the $P[x \succ y]$ probabilities as transition probabilities which govern the evolution of the PCE over a set of two or more options, in the spirit of the hypothetical negotiation process supporting the Median Voter Theorem. The PCE does not rely on any particular function

for the winning probability. In particular, if the probability is 100 percent that the stronger coalition wins, no matter how small the margin, then the PCE replicates the standard Deterministic Condorcet Election. Thus, the CDMPs leading to both the median voter and the Central Position are special cases of a PCE, and PCE’s are a proper superset of Deterministic Condorcet Elections.

Since the result of each iteration of negotiation is only dependent on the previous iteration, the negotiation process is a Markov process. The limiting distribution of the Markov process gives the likelihood that each option is the winner at any given turn of the CDMP. For the trivial case of a state with just two options, $S=(x,y)$, with linear transition probabilities, the limiting distribution is exactly the intuitive result:

$$P[x | S] = \frac{s(x:y)}{s(x:y) + s(y:x)}$$

$$P[y | S] = \frac{s(y:x)}{s(x:y) + s(y:x)}.$$

For the case of two options, the Deterministic Condorcet Winner will have the stronger coalition and hence also the highest probability.

As with the original Condorcet Election, we compute the votes and coalition strengths for all pairs of options in the state S . The pairwise transition probabilities $P[x \succ y]$ are then used to compute the conditional probability for each option $P[x | S]$, and the option x with the highest probability is deemed the Probabilistic Condorcet Winner:

$$x = \arg \max_{\forall x \in S} P[x | S]. \tag{11}$$

The Probabilistic Condorcet Winner has important similarities to a standard Deterministic Condorcet Winner, as well as some useful differences. As shown above, the Deterministic Condorcet Winner and Probabilistic Condorcet Winner are identical for a choice between two options. For a wide variety of problems – one-dimensional spatial model, multidimensional spatial model, discrete combinatorial with random utilities, and so on – with proportional voting, the coalition structure can be computed so as to determine both the Deterministic Condorcet Winner and the Probabilistic Condorcet Winner. Almost always, the Probabilistic Condorcet Winner will be the Deterministic Condorcet Winner, i.e., the Central Position. There are still important differences, though:

Depending on the type of model and voting rule, a Deterministic Condorcet Winner might not exist, whereas the limiting distribution for the PCE Markov process will. This is true even when different probabilistic transition rules are allowed. (Several are implemented in the KTAB software distribution.)

Along with the winning option, which is a point estimate, the PCE provides an indication of the dispersion of plausible results around that estimate.

Symmetric negotiation using PCE

Under both the Median Voter and Central Position Theorems, note that office seeking politicians are supposed to iteratively propose new policies for voting that will incrementally lead to the weighted median position or Central Position. Here we describe one possible generic mechanism for determining new options by utility maximizing actors. This is similar to a symmetric version of the Baron-

Ferejohn model (Baron and Ferejohn 1989). The essential structure is that each utility maximizing actor has a utility for not just individual choices but also for an entire state, and each tries to maximize that utility.

We define the state of a CDMP during a given iteration as the set, S , of positions taken by the actors in that iteration:

$$S_t = (\theta_1, \theta_2, \dots, \theta_A).$$

Given the domain specific utility model on positions, $u_i(\theta)$, and a voting rule for each actor, we can build the strength of the coalition supporting each option in any given pair. As this is just another choice between two options, we construct coalitions and strengths exactly as before:

$$c(\theta_j : \theta_k) = \{i \mid v_i(\theta_j : \theta_k) > 0\}$$

$$s(\theta_j : \theta_k) = \sum_{i \in c(\theta_j : \theta_k)} v_i(\theta_j : \theta_k).$$

The simplest way to assign a utility to an entire state is to use the utility of its Central Position, which is a kind of Deterministic Condorcet Winner. This would produce a symmetric negotiation process with an embedded Central Position. However, decision-making processes in the real world are not precise and deterministic, so strategies such as ‘minimizing downside risk,’ or reducing the variance while holding the expected outcome the same, are important. Empirically, legislative coalitions often include a few more members than just the minimal winning coalition. A separate KTAB model of parliament formation suggests that this is a risk mitigation strategy: coalitions slightly compromise their goals to include more members. This reduces not only the harm to the coalition of defections but also the members’ incentive to defect from the coalition. Therefore, we chose to represent the observed risk management strategies by

implementing the symmetric negotiation process with an embedded PCE.

From the ratios of coalition strengths, we can compute Markov transition probabilities, which then gives the limiting distribution of the PCE over the set of options in the current state. We then define the value to the i^{th} actor of an entire state as the expected utility of the outcome of the state:

$$U_i(S_i) = \sum_{j=1}^N P[\theta_j | S] u_i(\theta_j)$$

If actor k were to change its position to α , we denote the new state with just one position changed as $S' = \sigma(k, \alpha | S)$. The expected utility of the new state is $U_i(S')$, where the PCE is repeated in the new state. These expressions can be combined to find the utility to the i^{th} actor of the new state: $U_i(\sigma(k, \alpha | S))$. This leads to a CDMP where each actor separately crafts a new position to maximize the expected utility to itself, given the support or opposition it expects from other actors:

$$\theta_{i,t+1} = \underset{\alpha \in \eta(\theta_{i,t})}{\operatorname{argmax}} U_i(\sigma(i, \alpha | S_t)). \quad (12)$$

Supporting or opposing a policy based solely on the utility expected if it were to be implemented is called ‘naïve voting.’ With proportional voting, a naïve voter would exert influence in a choice between two positions as follows:

$$v_i(\theta_j : \theta_k) = w_i [u_i(\theta_j) - u_i(\theta_k)],$$

thus, a naïve voter would exert the most effort to support an outcome they most prefer, compared with the alternative, and the least effort to support an outcome they did not greatly prefer.

Alternatively, strategic voting considers not just the value of an option but also the reaction of other

actors; the first level of strategic voting would use just the Central Position to evaluate a state. Because we implemented the CDMP with an embedded PCE, we included another level of strategy in voting. Under proportional voting, a strategic voter would exert influence as follows:

$$v_i(j, \alpha : k, \beta) = w_i [U_i(\sigma(j, \alpha | S)) - U_i(\sigma(k, \beta | S))].$$

In particular, the sign of $v_i(j, \alpha : k, \beta)$ would indicate whether actor i found it more strategically desirable for actor j to adopt position α or for actor k to adopt β , given the current state S . The magnitude of $v_i(j, \alpha : k, \beta)$ would indicate how much influence actor i would exert to get actors j or k to change their positions. These are essentially nested voting models: *naïve* voting is applied within a PCE to evaluate a state to determine its *strategic* value to an actor. In the ‘symmetric negotiation using the PCE CDMP, each actor uses this strategic evaluation of possible states to select their next position to advocate.

Strategic voting allows actors to assess not only the intrinsic desirability to themselves of a position, but also the support or opposition it is likely to receive from other actors. In an economic context, an individual actor would always want all the other actors to give enormous subsidies, but such a position would not win any support. Advocating a shortsightedly self-interested policy would likely fail and hence would have lower expected value than advocating a policy of enlightened self-interest that compromised just enough to win strong support from other actors with aligned interests. As a result, strategic voting can bring about seemingly counterintuitive behavior even in the simple economic models considered here. For example, an actor might support position x even though it is less desirable than the status quo, y , because doing so splits the coalition supporting y and increases the probability that a more desirable option, z , might win.

In other words, supporting x increases the expected value of the whole state, even though position x is less desirable than position y .

In principle, strategic voting could be applied by all the actors in a CDMP across states. Note that because it requires no special structure over the set of options or utility functions, the Central Position Theorem applies to a CDMP using strategic proportional voting over possible states. We simply define the set of options to be the (very large) set of possible states and the utility function to be expected utility from the PCE, and the Central Position Theorem automatically applies. This means that, with proportional voting, there is a ‘central state’ that the group will adopt over any other, and that the group has noncircular preference between whole states. A simple hill climbing CDMP to maximize ζ would lead to the ‘central state,’ but we will not discuss it further as it was not used in the Leontief tax/subsidy example (see the KTAB software distribution for examples which do use it).

The derivation of both the Median Voter Theorem and the Central Position Theorem relies on the presence of office seeking politicians to ensure convergence. With utility maximizing actors alone, there is no reason that they will ever propose the Deterministic Condorcet Winner if it is not in their interest. Hence, in theory, for arbitrary utility

functions and option sets, the symmetric negotiation with utility maximizing actors alone might cycle endlessly. However, in the real world, policy is formulated by a mixture of support maximizing politicians and interest maximizing pressure groups. Consequently, we ran the symmetric negotiation CDMP in two variants:

- Utility maximizing actors (UMAs) alone, which develop new positions purely to maximize the expected value to themselves of the next state.
- The same utility maximizing actors with some office seeking politicians. The office seeking politicians modify the UMAs’ proposals to develop positions with higher probability of being accepted in the next state.

In both variants, as with the Median Voter Theorem and Central Position Theorem, the probability of any proposal is determined by a PCE of the utility maximizing actors. Because of the structure of the economic models discussed below, both versions of the CDMP converge to similar results: a strategic equilibrium where each actor is doing the best it can, even anticipating the strategic choices of others. With complex domain specific utility models such as the economic models discussed below, it is common that the strategic equilibrium has the actors divided into clusters, where each cluster advocates a common policy.

Leontief Input-output Economic Model

So far, we have explained several generalizations of the standard academic framework for CDMPs – or, as it is generally referred to in the literature, bargaining – so that we can address more complex problems. We now describe a simple input-output model of a Leontief economy with both domestic demand and exports, in which there are N industrial sectors, M factors of production and a single consumption group.

There are natural economic linkages between the sectors, which can result in their promoting a common policy on taxes and subsidies. Because the value-added is counted for both sectors and factors, there are also natural economic linkages between them: an increase in value-added for unskilled labor used in the agricultural sector would be a benefit to both the agricultural sector and the unskilled labor factor, in general. There are $A=N+M$ actors, and the position of each actor is a vector τ of taxes and subsidies on N export goods. Domestic demand, D , is considered to be static, as are domestic prices, and demand is unaffected by the export taxation policy. The base year export quantities are the vector X_0 and the base year prices are P_0 ; for the i^{th} export commodity, the base year price $p_{i0} = 1$, as is standard in computable general equilibrium models. The vector elements have $\tau_i > 0$ for a tax and $\tau_i < 0$ for a subsidy. After applying the tax/subsidy vector, the price of the i^{th} export commodity is the following:

$$p_{i\tau} = 1 + \tau_i.$$

The vector of after-tax prices is P_τ , with corresponding after-tax export demands X_τ . Export demands depend nonlinearly on the policy; each export is constantly elastic in price for each commodity separately:

$$x_{i\tau} = x_{i0} \left(\frac{p_{i0}}{p_{i\tau}} \right)^{\epsilon_i}.$$

The tax/subsidy vector is constrained so the after-tax prices are no less than 1/10th and no more than 10 times the base year prices. It is further constrained to be revenue neutral (i.e., self-financing) after demands are adjusted for the new prices. This is a simple way to avoid the unrealistic solution of giving every actor the maximum possible subsidy. The set Ω of possible tax positions is therefore the following:

$$\Omega = \{\tau \mid \tau X_\tau = 0, -0.9 \leq \tau \leq 9\}$$

In line with the multiple economic views in Figure 1, we will now develop two distinct economic models of the same situation. Each model gives the economic expectations of actors, with no assumption that any view is necessarily the correct one.

The first economic view looks at equilibrium in a single period. Given a linear sub-model of how wages, interest, and government revenues are distributed to the consumption group, and their Cobb-Douglas utility functions, we can use basic linear algebra to calculate the $N \times N$ matrix, A , which specifies how total production depends on demand and exports:

$$\begin{aligned} Q &= AQ + \phi Q + X \\ &= [I - (A + \phi)]^{-1} X \\ &= L_\alpha X \end{aligned} \tag{13}$$

In (13), ϕ is the propensity of consumption for each of the factors of production. The basic Leontief model sets out that the amount of value-added for each factor f ($f=1,2,\dots,M$) per unit of production in a sector, s , ($s=1,2,\dots,N$) is a constant, ρ_{fs} . Thus, the $M \times N$ matrix of value-added, V , is a linear function of the $N \times 1$ vector of production. A simple component wise multiplication can represent this:

$$V_{fs} = \rho_{fs} Q_s,$$

which we abbreviate to $V=\rho Q$, where the indices are assumed.

In the Leontief model, the factors and sectors use different L matrices to estimate Q from the same X . This is the economic model that the factors use to estimate the effect on them of export tax policies, where V^F is an $M \times N$ matrix:

$$V^F(\tau) = \rho(L_\alpha X_\tau).$$

The value-added expected by factor f , given any policy of taxation/subsidization, is the sum over sectors for that factor:

$$v_f^F(\tau) = \sum_{s=1}^N V_{fs}^F(\tau). \tag{14}$$

To develop the second economic view, we extend the model to include the investment needed to support a constant, balanced rate of growth, g , over multiple periods. Again, our goal is to demonstrate integration of CDMP and economic models, rather than to reproduce sophisticated economic growth models: we treat growth as simply a constant percentage increase in output and capital stock. This changes the fundamental equation to include demand for investment goods, I :

$$Q = X + AQ + \phi Q + I.$$

In any given year, the capital required to produce all the output, B , cannot exceed the available capital stock, K , which means we must have $BQ \leq K$. Further, if we assume that the rate of depreciation, d , is constant across industries, we find the required vector of investment in each period required to support steady growth:

$$K_{t+1} = (1 - d)K_t + I_t = (1 + g)K_t.$$

This shows that the required investment is the proportion required to cancel depreciation and provide the desired growth: $I_t = (d+g)K_t$. Assuming no excess capital stock means that $BQ_t = K$, we can calculate the required level of investment as $I_t = (d+g)BQ_t$. This leads to the revised estimate of output, given export demand and expected investment:

$$\begin{aligned} Q &= X + AQ + \phi Q + (d + g)BQ \\ &= [I - (A + \phi + (d + g)B)]^{-1} X \\ &= L_\beta X \end{aligned} \tag{15}$$

This is the economic model used by the sectors to estimate the effect on them of export tax policies, using the same matrix as before. The resultant $M \times N$ matrix of value-added relationships is:

$$V^S(\tau) = \rho(L_\beta X_\tau).$$

The value-added expected by sector s , given any policy of taxation/subsidization, is the sum over factors for that sector:

$$v_s^S(\tau) = \sum_{f=1}^M V_{fs}^S(\tau). \tag{16}$$

The combination of the two simple economic models culminating in (14) and (16) form the domain specific utility model for the example CDMP in this paper.

Significantly, the CDMP does not have to assume that all the actors agree on the correct economic model. The possibility of actors having different

models of economy is both empirically plausible – watching a few minutes of television news will illustrate this – and mathematically permissible in both the Central Position Theorem and PCE frameworks. In practical terms, this is allowable because we are not trying to model what the actual consequences of a policy will be; we are trying to model how the decision-making process will proceed, even when actors disagree about possible future consequences. Where they agree that a policy increases overall gross domestic product, without loss to any actor, they would probably all favor implementing such a Pareto improvement, even if they do not agree on the exact value of the increase.

Because different actors can have different economic views, the policy outcome – either Central Position or Probabilistic Condorcet Winner – is not guaranteed to lie on the perceived Pareto frontier of any actor. It clearly cannot lie on the Pareto frontiers of all actors, because they have different frontiers. Again, it is not unusual in real economic policy formation for different actors to advocate, or tolerate, policies which are not quite optimal in their view, simply in order to win enough votes.

Risk-adjusted utility calibration

The raw utility to each actor from a position is the value-added that it expects from its particular economic model under that policy, but these must be calibrated in two ways. First, many psychological studies have shown that people tend to view potential losses of what they have as more significant than potential gains. This is relevant in this CDMP because as taxes are introduced, some

sectors will lose value-added from the base case. We model this asymmetric effect by converting the value-added numbers to ‘risk adjusted utility’ values: losses compared to the base year are weighted heavier than gains. The risk adjusted utility is a piecewise linear function of value-added, as illustrated in Figure 3. Note that any gains in value-added are directly translated, while losses are translated at an increased rate, giving lower utility.

This is expressed in the following formula; in our example, we use $r=3/2$.

$$u_i^*(\tau) = \begin{cases} v_i(\tau) & \text{if } v_i(\tau) > v_i(0) \\ r[v_i(\tau) - v_i(0)] + v_i(0) & \text{otherwise} \end{cases} \quad (17)$$

Second, note that we commonly normalize utilities to the [0,1] range, so that each actor’s influence stays in the $[-w_i^- + w_i^+]$ range. Because of not only the nonlinear constraints on allowable tax/subsidy policies, but also the complexity of the relations between tax/subsidy, demands, sectors, and factors in the Leontief model, it is impossible analytically to find a transformation that will normalize the values for $u_i^*(\tau)$. Instead, we use a stochastic normalization algorithm to give the final utility values:

1. Generate a large number of random but revenue neutral policies using Monte Carlo sampling.
2. Record the value-added expected by each actor, using the appropriate economic model, and transform to $u_i^*(\tau)$ using (17).
3. Measure the risk-adjusted utility in the worst case $u_i^*(\tau_{\min})$, and best case $u_i^*(\tau_{\max})$, for each actor.

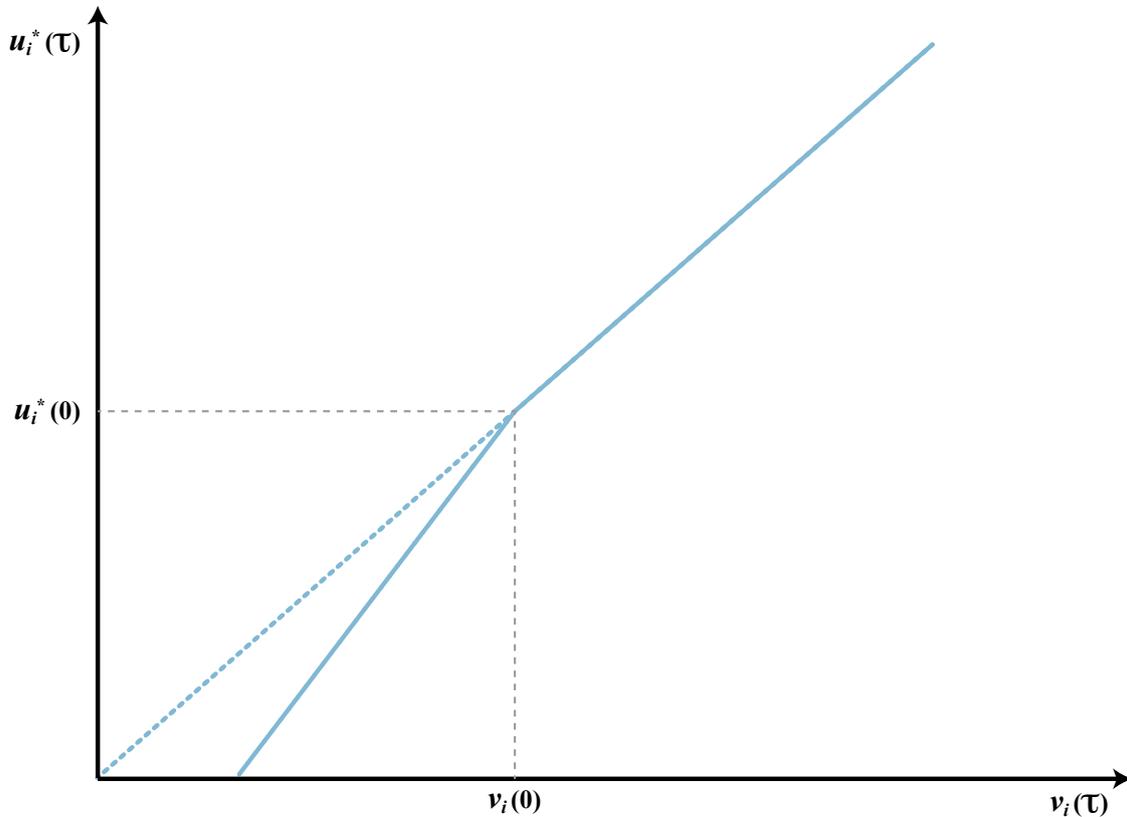


Figure 3. Risk adjusted utilities from value-added; the dotted 45 degree line shows the utilities if losses were treated symmetrically.

Source: KAPSARC.

4. This gives a rule to normalize each actor’s risk-adjusted utility to the von Neumann scale:

$$u_i(\tau) = \frac{u_i^*(\tau) - u_i^*(\tau_{\min})}{u_i^*(\tau_{\max}) - u_i^*(\tau_{\min})}$$

While the utility to each actor is a piecewise linear function of value-added, the value-added itself is not a linear function of the tax policy vector. Furthermore, it is clearly of dimension $N + M$, and is not single peaked as would be required by the Median Voter Theorem. However, it is still possible to verify that the value-added for each sector does show the expected decline as the export tax on

that sector increases. To demonstrate, Table 2 presents the correlations between the value-added to a sector, $v_i(\tau)$, and the export tax on that sector, τ_i , over a set of 2,500 samples. This table was created from some randomly generated data, the better to control the relationships between sectors and highlight the correlations. Thus, we are looking at the correlation of that actor’s value-added with just the corresponding component of the tax policy.

Most sectors show a strongly negative correlation as we would expect – increased taxation reduces value-added.

Leontief Input-output Economic Model

Table 2 does not show perfect linear correlations for three major reasons:

The export demand response to a tax rate is nonlinear, each to differing degrees.

Total value-added in a sector depends not only on its own direct export demand, but also on the domestic demand indirectly generated by export from other sectors. Those export demands depend on the other components of the tax vector.

Because each policy is revenue neutral, taxing one sector must provide subsidies to the other sectors, which have their own nonlinear demand curves. Taxing one export sector reduces export demand, but may drive up domestic demand. This effect is particularly strong for sector 4 in this dataset.

Table 2. Correlations of sectoral tax rate with sectoral value-added.

Sector	Correlation
1	-0.9264
2	-0.9629
3	-0.4183
4	-0.9362
5	-0.8805

Source: KAPSARC.

Numerical Results

Here we show numerical results with realistic data demonstrating two different CDMPs with the input-output economic models detailed in the previous section as the actors' utilities. The base case scenario is that there are currently no taxes or subsidies on exports from a set of industrial sectors. Actors are negotiating a policy of taxation/subsidization that is constrained to be revenue neutral. We first model the negotiation process among utility maximizing actors representing each sector and factor of production, proposing policies as self-interested utility maximizers. Each actor has its own separate position at each iteration of the CDMP, and they modify them in parallel as per (12). Secondly, we model the negotiation process followed by a set of office seeking politicians proposing policies as per (7), in which office seeking politicians propose iteratively better policies based on how likely they are to be accepted. Again, our goal is not to model a particular real situation, with reliable numeric values, but to illustrate a modeling methodology to generate solution policies which are reasonably plausible.

We synthesized the data from three sources to represent the U.S. economy. Our model considers

the $N=9$ sectors and $M=3$ factors of production to be 12 utility maximizing actors as shown in Table 3.

We began with U.S. aggregate economic data from 1981, sourced from the National Bureau of Economic Research Report by Auerbach (1988). We used the A and B matrices, as well as the values for d and g from Appendix B. This report did not have the base case value-added to the three production factors. As a surrogate, we obtained value-added data from 1976 for five industries in the U.S. state of Georgia, from Table 3.1 in Chapter 3 of Schaffer (2010). We aggregated the value-added from the five industries into the same sectors and converted them to share percentages. With these share percentages, it was trivial to sum the base case value-added by sector (matrix A in Auerbach (1988)), then compute the value-added from each sector into each factor. The A matrix and values-added shares are shown in Table 4. Lastly, the Auerbach article had neither domestic demand, nor export demand for the base year. We synthesized D and X_o by normalizing 2002 demand and exports by sector (U.S. Bureau of Economic Analysis, as cited in Miller and Blair 2009) from a total of \$9.3 trillion to the 1981 gross domestic product (GDP) of \$3.2 billion.

Table 3. Example sectors and factors.

Sectors	Factors
Agriculture (0.010)	
Mining (0.032)	
Construction (0.039)	
Durable goods manufacturing (0.071)	Household pay (0.239)
Nondurable goods manufacturing (0.043)	Other pay (0.117)
Transportation, communication and utilities (0.031)	Imports (0.144)
Wholesale and retail trade (0.071)	
Finance, insurance and real estate (0.089)	
Other services (0.114)	

Source: KAPSARC.

Numerical Results

Table 4. Leontief A matrix and factor value-added vectors showing actors' voting weights.

	AGR	MIN	CON	DUR	NDG	TCU	TRD	FIN	OTH
AGR	0.236	0.000	0.001	0.005	0.086	0.000	0.002	0.003	0.006
MIN	0.001	0.050	0.006	0.015	0.162	0.088	0.000	0.000	0.002
CON	0.011	0.037	0.002	0.008	0.007	0.032	0.008	0.033	0.019
DUR	0.018	0.039	0.292	0.351	0.034	0.026	0.008	0.002	0.036
NDG	0.163	0.023	0.064	0.064	0.296	0.105	0.040	0.011	0.098
TCU	0.029	0.024	0.030	0.049	0.058	0.177	0.063	0.022	0.055
TRD	0.041	0.012	0.079	0.052	0.045	0.021	0.020	0.004	0.031
FIN	0.065	0.048	0.014	0.016	0.012	0.023	0.063	0.146	0.053
OTH	0.022	0.018	0.095	0.042	0.044	0.050	0.134	0.057	0.098
Inputs	0.586	0.251	0.583	0.602	0.744	0.522	0.338	0.278	0.398
Value-added	0.414	0.749	0.417	0.398	0.256	0.478	0.662	0.722	0.602
HH pay	0.185	0.377	0.170	0.137	0.088	0.214	0.389	0.397	0.331
Other	0.091	0.166	0.049	0.060	0.039	0.105	0.223	0.207	0.173
Imports	0.138	0.206	0.198	0.201	0.129	0.160	0.051	0.118	0.099

Source: Auerbach 1988, Schaffer 2010, KAPSARC.

Note that, under proportional voting in (8), each actor votes – whether naïvely or strategically – between two options, based on the expected difference in utility scaled by an influence weight w_i . There are many considerations that could go into estimating these weights. In our example, we set the weight for each actor to be its total value-added in the base case. We chose to approximate influence by value-added because:

The ability to capture value-added in the past is a kind of ‘revealed capability’ for capturing value-added in the future.

Access to more monetary resources from current value-added provides more tools of influence for the future.

This is somewhat analogous to Negishi weights (Negishi 1927). While there are well-known normative problems with Negishi weights in the context of ‘value of human life’ for climate policy, the particular method of determining influence is not essential to the

demonstration of integration between policymaking and economic models. To emphasize the relative strengths of actors, the weights are shown in the parentheses in Table 3 after being scaled to sum to unity.

At the start of the first CDMP, every actor has the same status quo initial position: zero taxes on all sectors. The CDMP proceeds over several iterations, until all the positions stabilize. At each iteration, each actor separately finds a position for itself that maximizes the expected value to it of the resulting state. The search through potential positions is done by a simple hill climbing search algorithm which takes the best new vector it finds at each step of the way, until it reaches a local maximum, which is the actor’s new position for the next iteration. Each step of the search can be considered a simulation of that particular actor’s ‘backroom negotiations’ to win allies and build an advantageous coalition.

Each actor’s optimization takes into account the expected value of hypothetical states in which each

changes its own position, while assuming that all other actors do not change their positions. However because all actors are performing this optimization, other actors adjust their positions as well. Hence, the CDMP iterates until no actor changes significantly, i.e., until a Nash equilibrium is reached.

We ran the CDMP with three different scenarios of export price elasticity of demand, shown in Table 5. In the first scenario, the price elasticity is equal across all sectors. We arbitrarily chose the value of $\epsilon_i=2.5\%$, to simulate the general observation that prices are typically more elastic for imported goods. In the second scenario, we allowed the elasticities to vary by $\frac{1}{8}$ th between 2.0 percent and 3.0 percent by sector, with the sectors in descending order according to their value-added. In this scenario, the sector with the highest value-added, 'Other Services', also has the lowest price elasticity of demand. The sector with the lowest value-added, 'Agriculture', thus has the highest price elasticity. The third scenario is the exact opposite – price elasticity is higher for sectors with higher value-added.

We used these three scenarios to implicitly test the consistency of our model with economic understanding. Under scenario 1, a tax/subsidy will tend to decrease/increase export demand equally for every sector, since they all have the same elasticity. Because losses are weighted more heavily than gains – and no sector can demonstrate a distinct and unique advantage or disadvantage to having

a subsidy or tax – there should be no reason for the CDMP to converge to a policy that confers a differential benefit to any sector: the resistance would be stronger than the support. Therefore, we would expect the Condorcet Winner under this scenario to be the status quo. This is precisely what we observed in our model. At no point in the iterative process did an actor even propose a tax or subsidy to any sector.

Applying similar logic to the differential benefits, based on varying elasticity of demand, we would also expect the CDMP to converge to a Condorcet Winner demonstrating a consistent pattern under the remaining two scenarios. In both cases, economic understanding leads us to expect a policy in which sectors with the most elastic demand were granted the biggest subsidies, and the least elastic sectors taxed the most. This is because a sector subject to more elastic demand can negotiate from the strength of a demonstrable higher benefit relative to a sector where demand is less elastic. If the differences in elasticities produce differential gains large enough to overcome the heavier weighting of losses, the policy should be favored over the status quo. In addition, increased value-added in sectors directly impact the three factors of production through the input-output relations, so the factors can be expected to form coalitions with highly elastic sectors to further promote a policy of subsidization. The logic is that highly elastic sectors have a lot to gain from subsidies, while highly inelastic sectors have comparatively little to lose from taxes, so the winning

Table 5. Scenarios of price elasticity of demand per sector for exports.

	AGR	MIN	CON	DUR	NDG	TCU	TRD	FIN	OTH
1	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
2	3.0	2.75	2.625	2.25	2.5	2.875	2.375	2.125	2.0
3	2.0	2.25	2.375	2.75	2.5	2.125	2.625	2.875	3.0

Source: KAPSARC.

Numerical Results

coalitions consistently subsidize the elastic sectors and tax the inelastic, though the precise amounts depend on the economic details. While this is not guaranteed to maximize gross domestic product – and different actors will disagree on that question because they hold different economic expectations – it does maximize political support.

Note from (14) that the value-added for a given factor is summed across all the sectors. This has two consequences. First, the factors are stronger than the sectors. Second, the revenue neutrality of tax policies ensures that the stakes will be lower for factors than for sectors: gains (or losses) in one sector are always

somewhat balanced by losses (or gains) in other sectors. The net position of factors is therefore highly dependent on the details of the input-output models they use; there is no consistent pattern.

Under scenario 2, the CDMP reached Nash equilibrium in only two iterations, with all actors settling on remarkably similar proposed policies. Averaging the tax/subsidy proposed for each sector across all 12 proposals results in an average policy which has been plotted with respect to elasticity as the blue circles in Figure 4. Under scenario 3, the CDMP converged in three iterations, with the averaged results plotted as yellow squares.

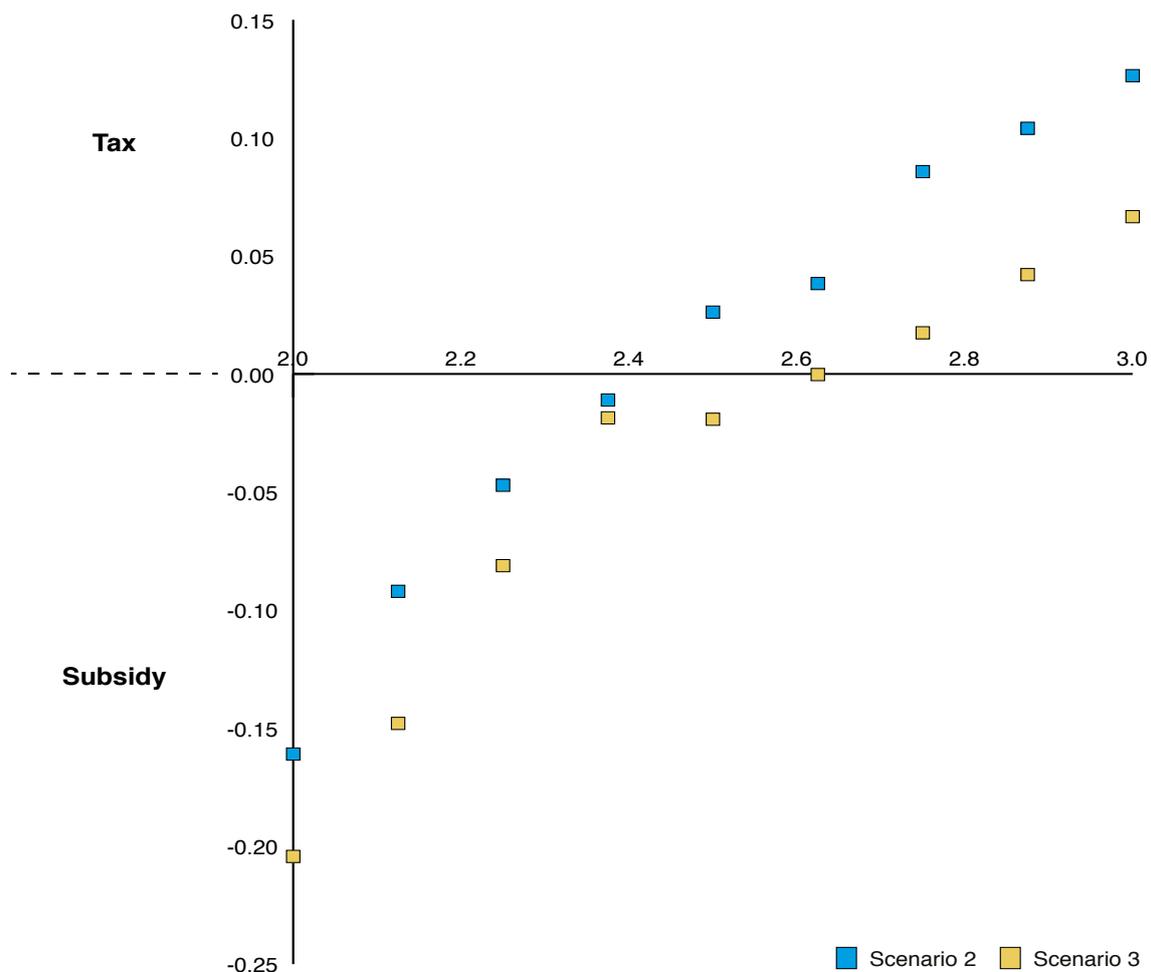


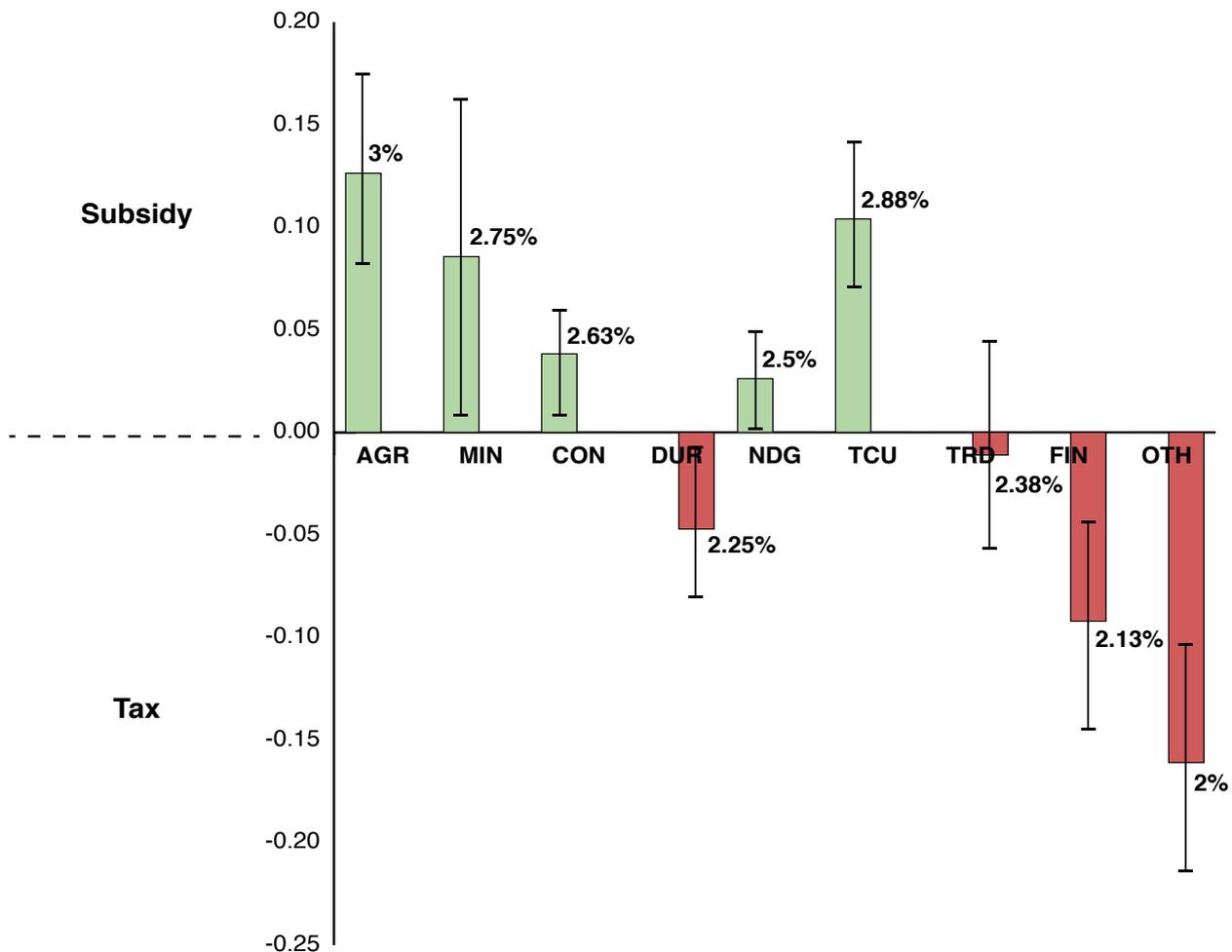
Figure 4. Demonstrating the correlation between elasticity and policy.

Source: KAPSARC.

Results for both scenarios show the expected positive correlations – 0.9843 and 0.9523 respectively. Note that we designed scenario 3 such that elasticity and value-added, and hence negotiation weight, were positively correlated. Since the sectors with the strongest differential advantage also have more negotiation power, we expected to see these sectors with stronger subsidies, as well as proportionally stronger taxes for the weaker sectors. Instead, we observe an unexpected shift to a higher level of taxation. Regardless of this, the fact that both scenarios result in a strong positive correlation between

elasticity and policy suggests that political negotiation based on economic benefit in a situation such as this is relatively robust to overly influential sectors.

Figure 5 shows the results from scenario 2. Figure 5(a) shows the average tax/subsidy for each sector, as plotted in Figure 4. The standard deviation around the mean for all 12 sectors and factors is also shown with error bars. We can see that within ± 1 standard deviation, all actors agreed on at least the direction of the policy, if not the magnitudes. The only exception was the Wholesale and Retail



(a)

Numerical Results

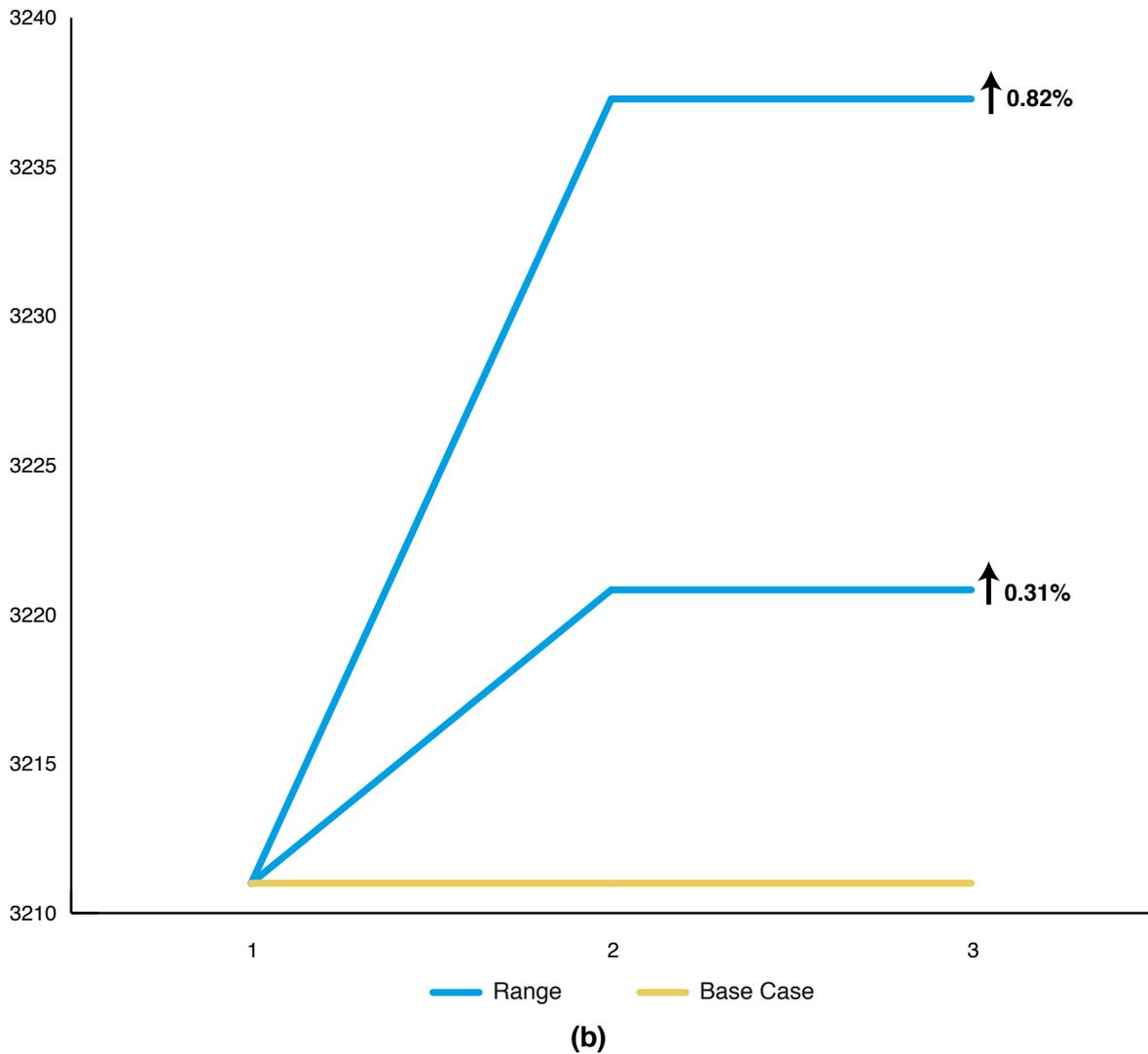


Figure 5. Results from scenario 2. (a): average policy of taxation/subsidization. (b): range of total GDP in each iteration.

Source: KAPSARC.

Trade sector, with middling price elasticity of export demand, about which the actors were almost evenly split between taxing/subsidizing.

In each iteration of the CDMP, the total GDP that would result from each proposed policy is computed. In Figure 5(b), total GDP, as computed using the factors' model (14), is plotted by iteration with the flat yellow line representing the base case GDP of \$3.2 billion. We can see that, even with a revenue neutral policy of taxation/subsidizing,

total GDP would be expected to increase by between 0.3 percent and 0.8 percent. Total GDP showed the same relative increase using the sector's model (16).

In our second CDMP, we modeled the negotiation process assuming the actors were office seeking politicians responsive to the same sectors and factors. The negotiation follows an iterative process of proposing policies as per (7), in which actors propose iteratively better policies based on how

likely they are to be accepted by utility maximizing actors. As with the Median Voter Theorem, the competition between office seeking politicians to be associated with the winning proposal drives the process to the Condorcet Winner, which is the Central Position. In each scenario of elasticities, the Central Positions are very similar to the Condorcet Winner arrived at by the group of utility maximizing actors. Under the second and third scenarios, the Central Positions showed the same pattern relating tax/subsidy levels to elasticity of demand (Figure 4). The correlation across sectors of the Central Position and the average policy from

the previous CDMP was 0.9938 for scenario 2 and 0.9887 for scenario 3.

The pattern of agreement between the two CDMPs – negotiation among utility maximizing actors alone, and negotiation among utility maximizing actors with office seeking politicians – is an interesting result in its own right. It suggests that the current practice of special interest lobbying, which results in a negotiation process similar to the second CDMP, can lead to the development of policies similar to what would have resulted if the actual special interests and other relevant actors had negotiated the policy themselves.

Conclusion

In this paper, we have demonstrated how a generalized quantitative model of informal CDMPs, based on the Probabilistic Condorcet Election, can be extended to use a realistic economic model for computing actors' utilities. A major advantage of basing this work on the PCE is that no restrictive economic assumptions are necessary. This integration allows us to overcome a major shortcoming of many models of policy and economy: that economic models typically treat policy choices as exogenously specified, while strategic models of policy negotiation, or CDMPs, typically use simple, fixed models of economics on which all actors must agree. In practice, political interactions and economic policies are entwined, as each may be affected by the details of the other. Further, the actors involved rarely agree on every detail of the economics of the situation. The methodology presented in this paper identifies ways to deal with both aspects of practical policy situations.

As a proof of principle, we have combined a fairly simple economic model with a relatively simple CDMP situation. There is no theoretical reason that this hybrid approach could not be applied to complex situations. A more realistic simulation of the functioning of an economy would simply require more complex economic models. However, we should note that negotiators generally rely on a simplified intuitive understanding of more complex economic models when they are actually negotiating. For example, if the question of the political economic factors likely to influence the scale and structure of a potential carbon emissions trading scheme were analyzed, the same PCE based framework as we integrate here with the simple Leontief economy could be integrated with a multiregional computable general equilibrium model. Future papers on this topic will explore the combination of more sophisticated, and more generally accepted, models of economic systems with our model of CDMPs.

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Notes

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About the Project

KAPSARC is developing the KAPSARC Toolkit for Behavioral Analysis (KTAB), an open source software platform, to support modeling and analysis of collective decision-making processes (CDMPs). KTAB is intended to be the standard platform for analyzing bargaining problems, generalized voting models, and policy decision-making. It is our intent to use KTAB to assemble the building blocks for a broad class of CDMPs. Typical models in KTAB will draw on the insights of subject matter experts regarding decision makers and influencers in a methodical, consistent manner; and then assist researchers to identify feasible outcomes that are the result of CDMPs.



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