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Commentary

A Decomposition Approach to Energy Policy Analysis

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Preeminent Decomposition Methods

Index decomposition analysis (IDA) has been used to study energy use movements since the 1980s. Methods at that time consisted primarily of multiplying factors that ultimately yielded the aggregate quantity, such as energy use (Ang and Zhang, 2000). IDA is a straightforward way for researchers to examine the individual impacts of economic intensity, economic structure, and other factors on energy consumption or greenhouse gas emissions. Other methods could take more complex approaches, such as analyzing the outputs of energy system models, to decompose aggregate quantities (e.g., Matar, 2016; Hirth, 2018).

IDA in energy economics has developed in the years since it was first introduced. Ang and Choi (1997), Ang et al. (1998), Ang and Zhang (2000), Ang and Liu (2001), Choi and Ang (2003), Ang et al. (2003), and Ang (2004, 2005) introduced and have continuously built on the logarithmic mean Divisia index (LMDI). Ang (2015) summarized the developments in and plethora of LMDI versions. There are multiple versions of LMDI, with the distinction being two different weights in the formulas, multiplicative or additive methods, or whether the analyst wants to explore energy in extensive or intensive terms.

The abovementioned methods use the same underlying mathematical formulation: an exponential of coefficients, or weights, multiplied by the logarithm of the multiplicative change in a particular factor. The weights are average logarithmic changes in the aggregate quantity or metric being decomposed. The factors are economic activity, changes in economic structure, energy intensity, or other factors. The appeal of these methods stems from the fact that although many factors are used, they produce a decomposition of the aggregate quantity that leaves no residuals. This decomposition property happens naturally through the formulations of weights and factor changes used.

This commentary discusses an alternative approach that incorporates prices and elasticities to decompose aggregate quantities and does not critique prevailing methods; rather, it presents decomposition from a different perspective.

Present Decomposition Method

Let us define energy use (*E*) as some function of income (*I*) and own price (*P*). Other factors (...) may and should also be considered. The statement in Equation 1 implies that *E* has an unknown functional form.

$$E = f(I, P, \dots) \tag{1}$$

The change in energy use (*dE*) may then be broken down into

$$dE = \frac{\partial E}{\partial I} dI + \frac{\partial E}{\partial P} dP + \cdots \tag{2}$$

The change in energy use (dE) may then be broken up into,

$$\boldsymbol{dE = \frac{\partial E}{\partial I} dI + \frac{\partial E}{\partial P} dP + \cdots}$$

Incorporating the definitions of income and price elasticities, E_I and E_P, respectively, into Equation 2 yields Equation 3:

$$\epsilon_I = \frac{\partial E}{\partial I} \cdot \frac{I}{E}$$

$$\epsilon_P = \frac{\partial E}{\partial P} \cdot \frac{P}{E}$$

$$dE = \epsilon_I \frac{E}{I} dI + \epsilon_P \frac{E}{P} dP + \cdots \tag{3}$$

As shown by Equation 4, integrating Equation 3 from some initial period (t_o) to a final period (t_f) yields the income and price effects from the change in energy use. In this step, the progressions of I and P coincide with that of time. That is, I_o happens at t_o, I_1 happens at t_1, I_2 happens at t_2, and so on.

As shown by Equation 4, integrating Equation 3 from some initial time period (t_o) to a final period (t_f) yields the income and price effects in the energy use change

Considering that $E \equiv f(I, P)$ and the elasticities are constants[1], the resulting income (IE) and price (PE) effects are shown by Equations 5a and 5b, respectively, which represent the result of numerical integration. A midpoint approach is taken for the integration, as depicted in Figure 1. As seen, the formulation can consider zero values for the elasticities, $\left(\frac{E}{I}\Big|_t + \frac{E}{I}\Big|_{t-1}\right)$, $(I - I_{t-1})$, $\left(\frac{E}{P}\Big|_t + \frac{E}{P}\Big|_{t-1}\right)$, and $(P - P_{t-1})$.

$$\int_{to}^{tf} dE = \underbrace{\int_{to}^{tf} \epsilon_I \frac{E}{I} dI}_{\text{Income effect}} + \underbrace{\int_{to}^{tf} \epsilon_P \frac{E}{P} dP}_{\text{Price effect}} \tag{4}$$

$$IE = \frac{\epsilon_I}{2}\left[\sum_{t_o+1}^{tf} \left(\frac{E}{I}\Big|_t + \frac{E}{I}\Big|_{t-1}\right) \cdot (I_t - I_{t-1})\right] \tag{5a}$$

$$PE = \frac{\epsilon_P}{2}\left[\sum_{t_o+1}^{t_f} \left(\frac{E}{P}\Big|_t + \frac{E}{P}\Big|_{t-1}\right) \cdot (P_t - P_{t-1})\right] \tag{5b}$$

There are two ways in which one can approach using Equations 5a and 5b. The first is incorporating some income and price elasticities from the literature. This method, however, does not guarantee perfect decomposition. The decomposition will have residual terms in the end.

The other way is to consider that elasticities have unknown values at the outset. An iterative solution could be obtained, where the elasticity values in Equations 5a and 5b are set to produce the actual change in aggregate quantity. In this case, the equations do not have residuals

[1] Constant elasticities over many years implies long-run behavior.

in the manner of LMDI methods. This method is preferred because it produces no residuals. An example is shown below.

An illustration: Electricity Use in Saudi Arabia from 2009 to 2019

A brief illustration is shown here for Saudi electricity use. Those data needed for the purpose of demonstrating the two equations are electricity use, average electricity price, and overall country gross domestic product (GDP). These data are shown in Table 1. GDP is used as a proxy for income since most income elasticities derived by statistical methods are obtained using GDP. GDP and price do not need to be deflated, as they are used in both the numerator and denominator in Equations 5a and 5b.

Figures 1 and 2 show $\frac{E}{I}$ and $\frac{E}{P}$ information, respectively, corresponding to Table 1. Figure 1 shows $\frac{E}{I}$ on the vertical axis and I on the horizontal axis because the integration is performed with respect to I. The calculations corresponding to Equation 5a are exemplified by the areas of the rectangles in Figure 1. Similarly, Figure 2 shows P on the horizontal axis. The horizontal axes in both graphs are arranged sequentially in time, from t_o to t_f, indicating that a certain time path was taken. Numerically integrating the above two curves produces the results of Equations 5a and 5b.

Table 1. GDP, electricity price, and electricity use in Saudi Arabia from 2009 to 2019 (sources: the Saudi Electricity & Co-generation Regulatory Authority (ECRA), the Saudi General Authority for Statistics (GaStat), and the Saudi Central Bank (SAMA)).

	I (million SAR)*	*P (SAR/MWh)*	*E (TWh)***
2009	1,885,745	123.00	193.47
2010	1,980,777	131.00	212.26
2011	2,198,539	141.00	219.66
2012	2,317,863	141.00	240.29
2013	2,383,930	141.00	256.69
2014	2,479,946	138.00	274.50
2015	2,596,259	138.00	286.04
2016	2,657,611	163.00	287.69
2017	2,655,758	157.00	288.66
2018	2,729,117	205.72	289.82
2019	2,751,831	210.47	279.68

*SAR = Saudi Arabian riyals
** The raw data for *E* are in MWh.

Figure 1. $\frac{E}{I}$ for Saudi electricity demand using the data in Table 1.

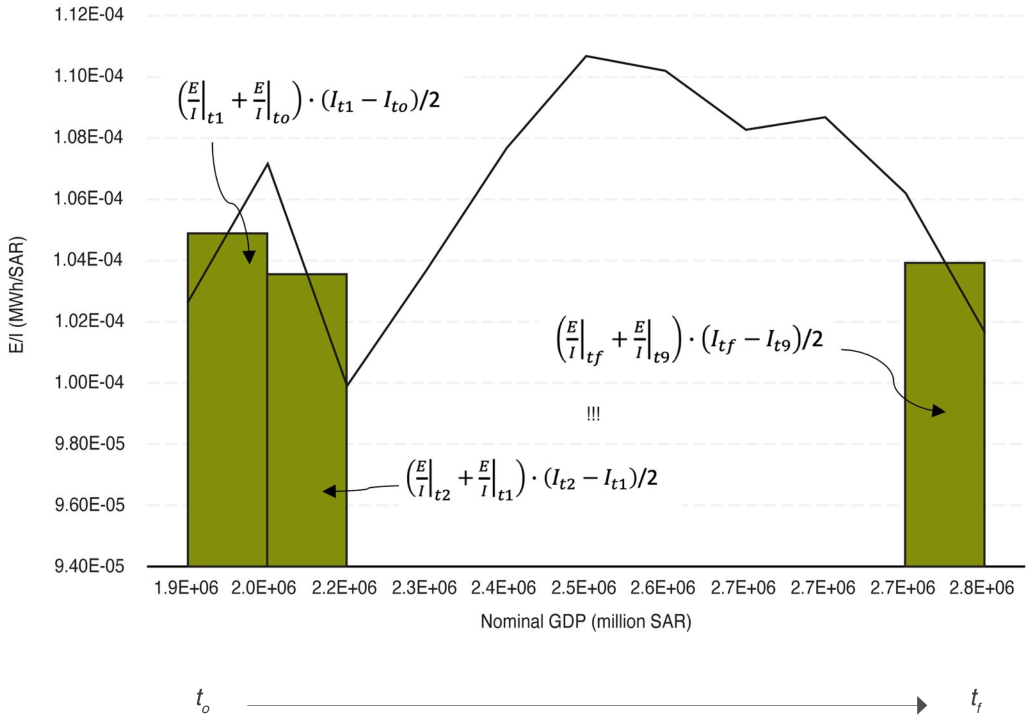
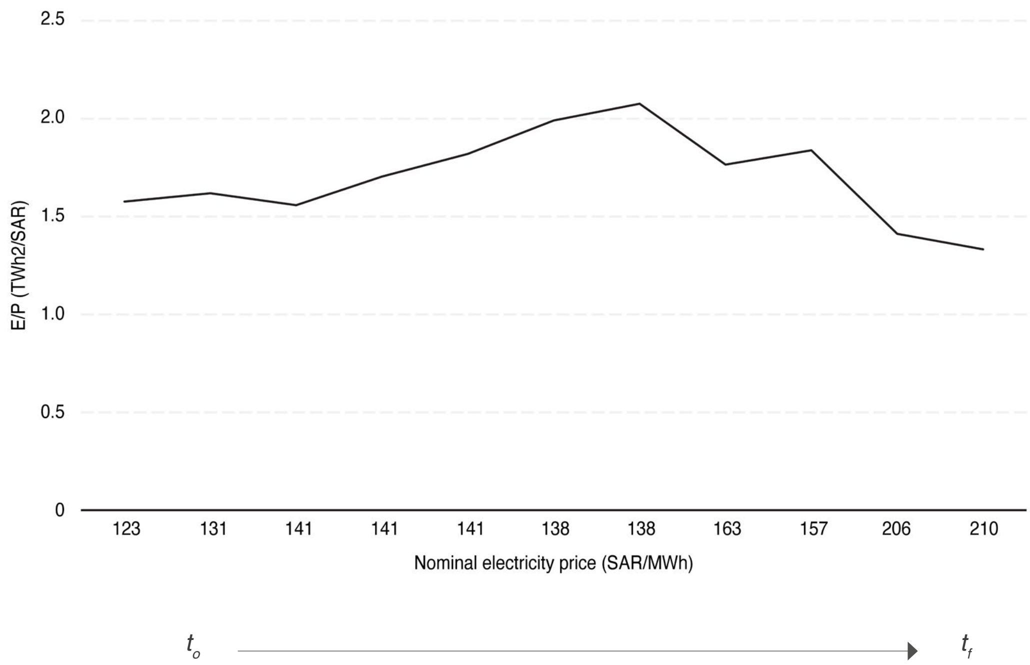


Figure 2. $\frac{E}{P}$ for Saudi electricity demand using the data in Table 1.



E_I and E_P are allowed to vary independently until the absolute value of the difference between the resulting and actual energy use divided by the actual change is less than 10^{-4}%. Over two million pairs are used for income elasticities that range from 0 to 2 and price elasticities that range from -1 to 0. The results for Equations 5a and 5b and their annual terms are shown in Table 2. The ensuing E_I and E_P pairs that satisfy the convergence tolerance are shown at the bottom of Table 2.

Table 2. Income and price effects for Saudi electricity use (source: author's analysis).

	$\left(\frac{E}{I}\Big\|_t + \frac{E}{I}\Big\|_{t-1}\right) \cdot (I_t - I_{t-1})$	$\left(\frac{E}{P}\Big\|_t + \frac{E}{P}\Big\|_{t-1}\right) \cdot (P_t - P_{t-1})$	
2010	19.93	25.55	
2011	45.09	31.78	
2012	24.29	0.00	
2013	13.96	0.00	
2014	20.97	-11.43	
2015	25.69	0.00	
2016	13.40	95.94	
2017	-0.40	-21.62	
2018	15.76	158.22	
2019	4.72	13.01	
2010 to 2019	**Income effect (Equation 5a)**	**Price effect (Equation 5b)**	**Δ*E* (IE+PE)**
Case 1 ϵ_I = 1.154 ϵ_P = -0.134	105.84	-19.64	86.20623
Case 2 ϵ_I = 1.804 ϵ_P = -0.544	165.43	-79.23	86.20616

From the total change in Saudi electricity use, GDP growth has resulted in an electricity use rise of 105.84 TWh. The price effect has depressed electricity use by 19.64 TWh

In all, only two pairs of E_I and E_P in the vast domain of pairs analyzed satisfy the convergence criterion. The actual change is 86.20623 TWh, while the two cases yield an error of less than 10^{-4}%. Once Saudi-specific income and price elasticities are considered (e.g., Atalla and Hunt, 2016; Mikayilov et al., 2020; Aldubyan and Gasim, 2021), only Case 1 is broadly consistent with the estimates put forth in the literature. From the total change in Saudi electricity use, GDP growth has resulted in a rise in electricity use of 105.84 TWh. The price effect has depressed electricity use by 19.64 TWh. If more factors are added to income and price, then more possible cases may satisfy the convergence criterion.

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About the Project

This decomposition approach is part of work on the KAPSARC Energy Model (KEM). We developed KEM for Saudi Arabia to understand the dynamics of the country's energy system. It is a partial equilibrium model formulated as a mixed complementarity problem to capture the administered prices that permeate the local economy. KEM has been previously used to study the impacts of various industrial fuel pricing policies, improved residential energy efficiency on the energy economy, the feasibility of deploying power plant technologies in Saudi Arabia, and a way to computationally analyze residential electricity prices.

About KAPSARC

KAPSARC is an advisory think tank within global energy economics and sustainability providing advisory services to entities and authorities in the Saudi energy sector to advance Saudi Arabia's energy sector and inform global policies through evidence-based advice and applied research.

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