K-DSGE: A Dynamic Stochastic General Equilibrium Model for Saudi Arabia

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Summary

This paper describes a dynamic stochastic general equilibrium (DSGE) model of the Saudi Arabian economy, developed by KAPSARC researchers. The K-DSGE model is to be used for simulations and experiments to assess the impact of economic reforms within the Saudi Vision 2030 framework. The model will also complement the suite of models currently used at KAPSARC for macroeconomic analysis, to assess the impact of the Kingdom's public policies.

Dynamic general equilibrium models have become increasingly popular in energy economics research. They have been used to explore the macroeconomic effects of energy price shocks and energy policies such as domestic energy price reforms and the deployment of renewables. These models are micro-founded, internally consistent and designed to mimic empirical dynamics. DSGE models are commonly used to study the potential macroeconomic effects of structural reforms and stochastic shocks. They also capture the dynamic linkages between key macroeconomic variables.

To the best of our knowledge, DSGE models that represent the Saudi economy are not yet publicly available. The K-DSGE model is designed to represent the most relevant characteristics of the Saudi economy. It also provides a meaningful analysis of current and potential public policies. The model represents the Saudi economy as an oil- and gas-rich small open economy. The economy is populated by two representative households – Saudi and non-Saudi – and four representative firms producing tradable and non-tradable final goods, electricity and energy services. The model divides the Saudi labor force into Saudi nationals and expatriates.

The model includes three energy sources – oil, gas and renewable energy and includes sales taxes and taxes on labor and capital. In the model, the government maintains a balanced budget, collects tax revenues, provides public services and income transfers, employs Saudi nationals, and invests in oil, natural gas, and renewable energy production capacities. The government also exchanges a final good with the rest of the world, exports fossil fuels, and acquires foreign bonds. The domestic currency is pegged to the United States dollar within a fixed exchange rate regime. The model includes an oil price reaction function to capture the potential effects of the variation in global oil prices because of Saudi oil exports and the effects on Saudi Arabia's oil export revenues. The calibration procedure involves three types of parameters and exogenous variables: (i) parameters obtained from the literature, (ii) parameters independently calibrated and (iii) parameters jointly calibrated for the model to match some of the long-run relationships of the Saudi economy from 1987-2016. Four unanticipated shocks, estimated from the data, are considered as sources of economic fluctuations: (1) shocks to international oil and (2) gas prices and the total factor productivities in the (3) tradable and (4) non-tradable sectors.

To the best of our knowledge, this is the first DSGE model for Saudi Arabia that represents it as a resource-rich small open economy. It includes many features and is consequently complex. Nevertheless, the model is useful for analyzing individual policy changes or the effect of joint policy decisions.

Introduction

This paper describes a dynamic stochastic general equilibrium model for the Saudi Arabian economy developed by KAPSARC researchers. The model is to be used for simulations and exercises designed to assess the economic impact of alternative policy options within the general framework of the Saudi Vision 2030 plan.

Issues such as the impact of administered prices vis-à-vis liberalized markets, the value of a barrel of oil used domestically and saved for export, the opportunity cost of the domestic use of gas and oil are examples of applications on which the model will be put to work.

The model represents an oil and gas-rich small open economy with tradable and non-tradable goods (essentially services). The domestic currency is pegged to the United States (U.S.) dollar so that there is a fixed exchange rate regime. However, one possible use of the model is to assess the impacts of a devaluation.

The Saudi economy is rich in oil, an important share of which is exported. Because the country is an important player in the international oil market, the model includes an oil price reaction function.

Energy is a key variable for the Saudi economy. Domestically, oil and natural gas are used to generate electricity, to desalinize water, and to produce chemicals and refined products mostly for transportation.

An additional feature of the economy is the composition of the labor force between Saudi nationals and expatriates. These two groups differ, among other things, for the benefits and services they receive from the government. For example, Saudi nationals accounted for over 95% of total government employees in 2017 (SAMA 2018). The role of the government is therefore very important to the Saudi economy.

The prices for electricity, water and fuels are administered by the Saudi government below international prices. Recently, however, the government has increased these prices and introduced value added taxation.

This paper and its appendices of which they are an integral part describe the model, its detailed structure and its parametrization. Additional aspects concerning its dynamic response and some examples of policy experiments will be added later.

Key Features of Saudi Arabia's Economy

In 2017, Saudi Arabia's nominal gross domestic product (GDP) was equal to 683.8 billion U.S. dollars (US\$) or 1,773.5 in purchasing power parity (PPP) terms, ranking 19th (15th in PPP terms) globally (World Bank 2018). Per capita nominal GDP was equal to US \$20,761 and US \$53,845 in PPP terms per capita. Per capita gross national income in 2017 was equal US \$20,080 ranking 37th and US \$54,770 in PPP terms per capita, ranking 13th.

In 2017, the Saudi population was 20.4 million, 12.2 million of whom (37% of the total population) were non-Saudi nationals. At the beginning of 2016, the unemployment rate among Saudis was 12.1%. In 2016, 67% of the workforce (excluding non-Saudi nationals) was employed by the public sector. In 2015, wages in the public sector were on average 1.5 times higher than those in the private sector.

Saudi Arabia is the world's second-largest holder of proved oil reserves, after Venezuela, holding roughly 16% of total global reserves. During 2017, Saudi Arabia was the largest exporter of crude oil, with a share of 15.9% of total crude oil exports. In the same year, Saudi Arabia was also the world's second-largest petroleum liquids producer behind the United States (U.S.), and the world's second-largest crude oil and lease condensate producer behind Russia.

Saudi Arabia is the fourth-largest holder of proven natural gas reserves in the world behind Russia, Iran and Qatar. Most Saudi natural gas fields are associated with petroleum deposits which are found in the same wells as crude oil. Increased production of this type of natural gas, therefore, remains linked to an increase in oil production.

Saudi Arabia's economy is heavily dependent on oil exports, which accounted for nearly 75% of total Saudi export value in 2016, representing about 60% of the Saudi government's revenues. The oil sector accounts for about 44% of the country's GDP, while the private sector accounts for about 40%. Real GDP growth largely depends on oil: In 2016 Saudi Arabia's oil-driven growth significantly slowed as a result of the decrease in crude oil prices since mid-2014, and oil revenues have fallen dramatically, notwithstanding the increase in oil output.

Saudi Arabia was the world's 10th largest consumer of total primary energy in 2016 at 266.5 million tonnes of oil equivalent (toe), of which about 63% was crude oil and petroleum liquids-based. Natural gas accounted for nearly all of the remaining 37% of consumption. Domestic consumption growth has been spurred by economic expansion in the wake of historically high oil-related revenues that persisted until mid-2014. In addition, administered low fuel prices, which reportedly cost the Saudi government an estimated \$61 billion in 2015, led to demand growth of about 7% per year between 2006 and 2016. Despite crude oil revenues falling from historic highs in 2014, Saudi Arabia's energy consumption grew steadily, mainly as a result of government subsidies (EIA 2017).

Saudi Arabia generated 330.5 billion kilowatthours (kWh) of electricity in 2016 with generation capacity of approximately 66 gigawatts (GW) (a good account of the energy system of the Gulf Cooperation Council members is given by Wogan, Pradhan, and Albardi [2017]; additional information reported here draws from EIA [2017]). The country faces a sharply rising

Key Features of Saudi Arabia's Economy

demand for power. This is driven by population growth, a rapidly expanding industrial sector led by the development of petrochemical industries, high demand for air conditioning during the summer months, and heavily subsidized electricity rates. Nearly all of the existing generating capacity is powered by oil or natural gas, but Saudi Arabia plans to diversify fuels used for generation, in part to free up oil for export. It also plans to deploy 27.3 GW of renewable energy, mostly solar photovoltaic (PV) generation and wind technology, by 2023.

Saudi Arabia's oil and natural gas operations are dominated by Saudi Aramco, the national oil and gas company, and the world's largest oil company in terms of production. The Saudi Electricity Company (SEC) is the largest provider of electricity in Saudi Arabia, with total available generation capacity of 74.3 GW. An SEC subsidiary is responsible for the transmission and distribution of electrical power. The state-owned Saline Water Conversion Corporation (SWCC) provides most of Saudi Arabia's desalinated water and is the second-largest generator of electricity, at 7.4 megawatts (MW) of installed capacity.

There is no personal income tax in Saudi Arabia. However, nonresidents conducting business in the Kingdom are taxed as corporations. The corporate income tax rate is 20%. The rate on corporations working in the natural gas sector is 30%. All Saudi

resident individuals and owners of businesses are obliged to pay 'zakat,' or religious/charity tax of 2.5%, based on net income, including all revenue, profit and capital gain. An indirect value added tax (VAT) of 5% was introduced on January 1, 2018, and applies to almost all supplies of goods or services, subject to limited exceptions.

In 2016, Saudi Arabia announced Saudi Vision 2030, a national transformation reform plan encompassing cultural, governance, and economic objectives (Government of Saudi Arabia 2016). The plan aims to reduce Saudi Arabia's dependence on oil, diversify its economy, and develop public service sectors such as health, education, infrastructure, recreation and tourism. Goals include reinforcing economic and investment activities, increasing non-oil industry trade between countries through goods and consumer products, and increasing government spending on the military and manufacturing equipment. The plan outlines far-reaching reforms of the energy sector and includes the partial privatization of state-owned Saudi Aramco. Income from Saudi Aramco's initial public offering would be used to help finance the country's economic transition. In addition, the government intends to set up a US\$ 2 trillion sovereign wealth fund, by far the world's largest. The income generated would provide an alternative to oil revenues and would help to realize the ambitions of Vision 2030.

Literature Review

In recent years, there has been considerable interest in developing dynamic stochastic general equilibrium (DSGE) models that are better designed to mimic empirical dynamics, study the potential macroeconomic effects not only of structural reforms but also of stochastic shocks, and capture the dynamic linkages between the main macroeconomic variables. This modeling approach represents a useful tool for macroeconomic evaluation and policy analysis in an environment which integrates several nominal and real frictions into a fully micro-founded setup. Recent critical evaluations of DSGE models are provided by Christiano, Eichenbaum, and Trabandt (2018) and Lindé (2018).

Based on the work of the ‘rational expectations’ school, or ‘new classical macroeconomics,’ early DSGE models described economies where fluctuations in aggregate economic activity are an efficient response of the economy to exogenous technology shocks in agents’ environments. The implied policy implications of those early models were that there is no need for any form of government intervention. Those ‘real business cycle’ (RBC) models incorporated the idea that monetary policy is ineffective for real variables. Early examples include Kydland and Prescott (1982) and Long and Plosser (1983).

However, empirical evidence soon revealed the limits of some key assumption underlying RBC models, such as perfect credit and insurance markets, and perfectly frictionless markets. In addition, those models had difficulty in accounting for some key properties of the aggregate data, such as the observed volatility in hours worked, the equity premium, the low co-movement of real wages and hours worked or – in open-economy versions of these models - the cyclical co-movement of consumption and output across countries.

The new prototypical DSGE model incorporated nominal frictions in goods and labor markets, such as sticky prices and wages, into the basic RBC model (Calvo 1983; Christiano, Eichenbaum, and Evans 2005). In these so-called ‘new Keynesian’ DSGE models, monetary policy has essentially no impact on real variables like output and the real interest rate in the long run. However, due to sticky prices and wages, monetary policy matters in the short run (Yun 1996; Clarida, Gali, and Gertler 1999; Woodford and Walsh 2005).

After the seminal papers by Smets and Wouters (2003, 2007) and Christiano, Eichenbaum, and Evans (2005), which have shown how the DSGE modeling approach can be used as a tool for policy analysis, a number of central banks, ministries and international institutions have developed their own DSGE models or have planned to do so in the near future (Edge et al. 2010; Sbordone et al. 2010). Note that DSGE models have been blamed for their inability to predict the 2007-2008 financial crisis. While this discussion goes beyond the scope of this project, one can refer to Christiano, Eichenbaum, and Trabandt (2018) and the references therein to form an opinion. Similarly, the financial crisis itself has prompted intensive research efforts to incorporate different types of financial frictions in post-financial crisis DSGE models and to enable them to assess the implications of unconventional monetary policy and of zero lower bound situations. See Christiano, Eichenbaum, and Trabandt (2018) for a short account of these and other DSGE developments. These aspects go beyond the scope of the present effort and are not elaborated upon here. In a similar vein, we do not elaborate on how DSGE models are estimated and their quantitative properties evaluated, even though it is an important and quite technical issue (see Sbordone et al.

[2010], for an example and Christiano, Eichenbaum, and Trabandt [2018], for a short discussion and further references).

The model described here shares the neoclassical features of many other models, where output is determined by technology, preferences and the supply of factor inputs (capital, labor and energy sources). In its current version, the model accounts only for real frictions in the neoclassical tradition. Its stochastic version allows for productivity shocks and shocks to energy (oil and gas) prices. Future extensions of the model may account for features common to conventional new Keynesian models, including a large variety of nominal and real frictions affecting the short- and medium-term behavior of the economy.

Dynamic general equilibrium models have become increasingly popular in energy economics research. They have been used to explore the macroeconomic effects of energy price shocks (Kim and Loungani 1992; Rotemberg and Woodford 1996; De Miguel et al. 2009) and to evaluate the impact of energy policies on the business cycle (De Miguel and Manzano 2006). General equilibrium models have also been used to study the asymmetric impact of oil subsidies on oil exporting countries and oil importing countries (Plante 2014) or the macroeconomic effects of removing fossil fuel subsidies (Lin and Li 2012; Schwanitz et al. 2014). Finally, Blazquez et al. (2018) analyze the implications of the deployment of renewable energy in Saudi Arabia, with its highly subsidized fossil fuels, using simulations from a dynamic general equilibrium model.

A recently developed area of the literature incorporates environmental externalities in DSGE

models. Chang et al. (2009) study optimal taxes on labor, capital income and pollution and their impact on the business cycle in a model with market imperfections. Among other things, these environmental dynamic stochastic general equilibrium (E-DSGE) models advocate optimal policies that dampen the procyclicality of emissions in response to a business-cycle shock (Heutel 2012; Annicchiarico and Di Dio 2015). Fischer and Springborn (2011), Annicchiarico and Di Dio (2015), and Dissou and Karnizova (2016) study different emissions mitigation policies, including cap-and-trade and carbon taxes. Annicchiarico and Di Dio (2017) and Holladay, Mohsin, and Pradhan (2018) study the international transmission of shocks and the emissions leakage following unilateral mitigation policies. Finally, Annicchiarico and Di Dio (2017) and Economides and Xepapadeas (2018) give the role of monetary policy special consideration in E-DSGE models.

Several studies in the literature have focused on Saudi Arabia without using a general equilibrium approach. Among these, Alyousef and Stevens (2011) discuss the costs of administered energy prices to the Kingdom's economy. Matar et al. (2015) use a multi-sector equilibrium model with a mixed-complementarity formulation to consider different policy scenarios for reducing energy consumption without increasing administered energy prices. They also analyze the prospects for coal-fired power generation. Using an econometric approach, Gately, Al-Yousef, and Al-Sheikh (2012) assess the future evolution of oil consumption and its impact on Saudi oil exports, while Pierru and Matar (2014) explore the impact of volatile oil revenues on public investment.

An Overview of K-DSGE

The model describes a small open economy taking as given the world interest rate, world prices of tradable goods and the international prices of oil and gas. The national currency is pegged to the U.S. dollar. There are two representative households – Saudi and non-Saudi – and four firms producing a tradable and non-tradable final good, energy services and electricity. There is a resource sector with three energy sources – oil, gas, and renewable energy. Foreign exchange consists of trade in a final good, exports of fossil fuels, and net acquisitions of foreign bonds. Finally, there is a fiscal authority potentially adopting revenues and spending decisions. The model features a tax system with sales taxes and income taxation from labor and capital. In addition to collecting revenues, the government provides public services and income transfers, employs Saudi nationals, and invests in oil, natural gas, and renewable energy production. Figure 1 is a graphical representation of the model structure.

The structure of the model consists of a standard neoclassical general equilibrium model, which in its current version includes one type of real friction, capital adjustment costs, enabling the model to capture the volatile behavior of investment and mimic its empirical dynamics in response to shocks. The current version of the model accounts for four types of shocks – productivity shocks in tradable and non-tradable goods production and shocks to oil and natural gas prices.

We outline below the detailed behavior of the different types of agents, the resource sectors, the government budget constraint, and characterize the competitive equilibrium and the economy's aggregate resource constraint. Appendix I provides the list of notations, variables and parameters in the model; Appendix II details the derivations of all the equations in the model, and Appendix III summarizes the model's equations.

Figure 1. K-DSGE model structure

Source: KAPSARC.

Structure of the Model

5.1 Households

We represent the Saudi economy using two types of representative households: Saudi and non-Saudi. So, the total labor force (N) in the economy is the sum of the Saudi and non-Saudi labor forces as denoted by:

$$(1) \quad N_t = N_t^s + N_t^{ns} = (1 - xpat) * N_t + xpat * N_t$$

where, $xpat$ is the share of the non-Saudis in the total labor force; we assume the share to be constant in our model. The subscript 's' denotes the Saudi household and the subscript 'ns' denotes the non-Saudi household.

5.1.1 The Saudi household

The Saudi household maximizes an expected discounted flow of utility subject to its budget constraint. The household's utility depends upon the consumption of a final good, energy services, leisure and public services. The final consumption good is a composite of tradable and non-tradable goods. Energy services include air conditioning, the use of electrical appliances, private transportation, and the like, while public services include government-provided healthcare, education, and national defense, among other services.

The budget constraint includes spending on the final (tradable and non-tradable) consumption good, investment goods and energy services, together with the net acquisition of both government and foreign bonds. Income includes after-tax labor income, capital income and government transfers as well as returns from the government and foreign bonds.

Thus, the Saudi household's problem is to maximize its welfare, given by the expected present discounted value of an infinite stream of

instantaneous utilities:

$$(2) \quad E_0 \sum_{t=0}^{\infty} \beta^t U^s(C_t^s, ES_t^s, l_t^s + l_{g,t}^s, GC_t)$$

where, β is the discount factor, C is the final consumption good, ES is energy services, l is labor supply to the private sector (tradable and non-tradable goods sectors), l_g is labor supply to the public sector, and GC is government consumption. E_0 is the conditional expectation operator. We assume labor supplied to the private and public sectors are perfect substitutes. In addition, we assume that only Saudi households work in the public sector and the supply of labor in the public sector is exogenous. The final consumption C is given by a constant elasticity of substitution (CES) aggregator, that also applies to both the Saudi and non-Saudi households:

$$(3) \quad C_t = \left[\theta_c^{\frac{1}{\sigma_c}} C_{T,t}^{\frac{\sigma_c-1}{\sigma_c}} + (1 - \theta_c)^{\frac{1}{\sigma_c}} C_{NT,t}^{\frac{\sigma_c-1}{\sigma_c}} \right]^{\frac{\sigma_c}{\sigma_c-1}}$$

where, C_T and C_{NT} are tradable and non-tradable consumption goods, respectively. θ_c is a share parameter of tradable consumption goods in the aggregate. σ_c is the elasticity of substitution between the tradable and non-tradable consumption goods. Note that if $\sigma_c=1$, we will have a Cobb-Douglas aggregator.

C_T and C_{NT} are defined respectively, by:

$$(4) \quad C_{T,t} = \theta_c \left[\frac{(1+\tau_{T,t})P_{T,t}}{(1+\tau_{C,t})P_t} \right]^{-\sigma_c} C_t$$

$$(5) \quad C_{NT,t} = (1 - \theta_c) \left[\frac{(1+\tau_{NT,t})P_{NT,t}}{(1+\tau_{C,t})P_t} \right]^{-\sigma_c} C_t$$

where, τ_T and τ_{NT} are consumption taxes on the tradable and non-tradable consumption goods, respectively. τ_c is the equivalent consumption tax on aggregate goods. P_T and P_{NT} are prices before taxes of tradable and non-tradable goods, respectively. P is the before tax price of the final consumption good, defined by:

$$(6) \quad P_t = \{\theta_c P_{T,t}^{1-\sigma_c} + (1-\theta_c) P_{NT,t}^{1-\sigma_c}\}^{\frac{1}{1-\sigma_c}}$$

In the case of a Cobb-Douglas aggregator for consumption ($\sigma_c=1$), we obtain:

$$P_t = P_{T,t}^{\theta_c} P_{NT,t}^{1-\theta_c}$$

This expression is what we define as the producer price index (PPI). Adding consumption tax we obtain the corresponding consumer price index (CPI), which, in the case of the CES aggregator:

$$(1+\tau_{c,t})P_t = \{\theta_c((1+\tau_{T,t})P_{T,t})^{1-\sigma_c} + (1-\theta_c)((1+\tau_{NT,t})P_{NT,t})^{1-\sigma_c}\}^{\frac{1}{1-\sigma_c}}$$

See Appendix A.27 for the derivation of $\tau_{c,t}$. In the case of the Cobb-Douglas aggregator:

$$(1+\tau_{c,t})P_t = ((1+\tau_{T,t})P_{T,t})^{\theta_c} ((1+\tau_{NT,t})P_{NT,t})^{1-\theta_c}$$

The supply of labor (l) is a CES aggregate of labor supply in the tradable and non-tradable goods sectors. This applies to both the Saudi and non-Saudi households. The supply of labor is given by:

$$(7) \quad l_t = \left[\theta_L^{\frac{1}{\sigma_L}} l_{T,t}^{\frac{\sigma_L-1}{\sigma_L}} + (1-\theta_L)^{\frac{1}{\sigma_L}} l_{NT,t}^{\frac{\sigma_L-1}{\sigma_L}} \right]^{\frac{\sigma_L}{\sigma_L-1}}$$

where, l_T and l_{NT} are labor supplied to the tradable and non-tradable goods sectors. θ_L is the share parameter of labor supplied to the tradable goods

sector. σ_L is the elasticity of substitution between the supply of labor to the tradable and non-tradable goods sectors. l_T and l_{NT} are defined by:

$$(8) \quad l_{T,t} = \theta_L \left[\frac{W_{T,t}}{W_t} \right]^{-\sigma_L} l_t$$

$$(9) \quad l_{NT,t} = (1-\theta_L) \left[\frac{W_{NT,t}}{W_t} \right]^{-\sigma_L} l_t$$

where, W_T and W_{NT} are nominal wages in the tradable and non-tradable goods sectors, respectively. Note that, since we assume the same tax rates (τ_w) on wages in the tradable and non-tradable goods sectors, the tax rate also equals the tax on the aggregate wage (W).

The Saudi household's budget constraint is:

$$(10) \quad (1+\tau_{c,t})P_t C_t^s + P_t \left[I_{T,t} + \frac{h_T}{2} \left(\frac{I_{T,t}}{K_{T,t-1}} - \delta_T \right)^2 K_{T,t-1} \right] + P_t \left[I_{NT,t} + \frac{h_{NT}}{2} \left(\frac{I_{NT,t}}{K_{NT,t-1}} - \delta_{NT} \right)^2 K_{NT,t-1} \right] + (1+\tau_{ES,t})P_{ES,t}ES_t^s + x_t B_t^* - (x_{t-1} + x_t i_t^*)B_{t-1}^* + B_t - (1+i_t)B_{t-1} = (1-\tau_{W,t})[W_t^s l_t^s + W_{g,t}^s l_{g,t}^s] + (1-\tau_{K,t})[i_{T,t} K_{T,t-1} + i_{NT,t} K_{NT,t-1}] + TR_t$$

where, I is gross investment, K is the stock of capital, δ is the depreciation rate on the stock of capital, h is the adjustment cost of the investment parameter, W_g^s indicates wages in the public sector, τ_{ES} is the consumption tax on energy services, P_{ES} is the price of energy services and ES is energy services. We allow a quadratic adjustment of costs in fixed capital accumulation. The budget constraint includes government transfers (TR) and the household's net acquisition of government (B) and foreign bonds (B^*). The latter stock is denominated in U.S. dollars (US\$), so that x is the nominal exchange rate between Saudi riyals (SAR) and US\$, whereas i_T , i_{NT} are the nominal returns on capital stock for the tradable and non-tradable sectors, respectively. i is the interest rate on government bonds and i^* is the interest rate on foreign bonds. Following Schmitt-Grohé and Uribe (2003), the foreign interest rate is governed by the debt-elastic interest rate:

Structure of the Model

$$(11) \quad i_t^* = i^{**} + \zeta[\exp(B_{ss}^* - B_{t-1}^*) - 1]$$

where i^{**} is an exogenous foreign interest rate determined in the foreign capital markets. ζ is the risk-premium parameter and B_{ss}^* is the steady-state level of the foreign bond. Note that we induce stationarity in the model by assuming that the interest rates of foreign bonds depend on the level of external debt, which can be interpreted as a transitory change in the sovereign risk premium. The capital accumulation is given by:

$$(12) \quad I_{T,t} = K_{T,t} - (1 - \delta_T)K_{T,t-1}$$

$$(13) \quad I_{NT,t} = K_{NT,t} - (1 - \delta_{NT})K_{NT,t-1}$$

The solution of the above constrained welfare maximization program yields optimality conditions for the following decision variables $C_t^s, B_t^*, B_t, ES_t^s, l_t^s$. Note that we take $GC_t, l_{g,t}^s, W_g^s$ and TR_t to be exogenous policy variables.

We parametrize the instantaneous utility function as the following:

$$(14) \quad U^s(C_t^s, ES_t^s, l_t^s, GC_t) = \ln\{[(C_t^s)^\sigma + \kappa(ES_t^s)^\sigma]^{1/\sigma} + \alpha_{CG}GC_t\} - \left(\frac{\eta}{1+\nu}\right)(l_t^s + l_{g,t}^s)^{1+\nu}$$

where, κ is the weight of energy services in the CES aggregate of the final consumption good and energy services, σ is the intertemporal elasticity of substitution between the final consumption good and energy services, α_{CG} is the weight of government consumption, η is the weight of disutility from the supply of labor, and ν is the inverse of the Frisch elasticity of labor supply. For the consumption good, energy services and labor supply, the optimality conditions equating pairwise marginal rates of substitution to relative prices are:

$$(15) \quad \kappa \left(\frac{ES_t^s}{C_t^s}\right)^{\sigma-1} = \frac{(1+\tau_{ES,t})P_{ES,t}}{(1+\tau_{C,t})P_t}$$

$$(16) \quad \frac{\eta(l_t^s)^\nu \{[(C_t^s)^\sigma + \kappa(ES_t^s)^\sigma]^{1/\sigma} + \alpha_{CG}GC_t\}}{(C_t^s)^{\sigma-1} [(C_t^s)^\sigma + \kappa(ES_t^s)^\sigma]^{1/\sigma}} = \frac{(1-\tau_{W,t}^s)W_t^s}{(1+\tau_{C,t})P_t}$$

The following expressions give the intertemporal optimality conditions for government and foreign debt:

$$(17) \quad x_t \left(\frac{1}{(1+\tau_{C,t})P_t} \right) \left\{ \frac{(C_t^s)^{\sigma-1} [(C_t^s)^\sigma + \kappa(ES_t^s)^\sigma]^{1/\sigma}}{[(C_t^s)^\sigma + \kappa(ES_t^s)^\sigma]^{1/\sigma} + \alpha_{CG}GC_t} \right\} = \beta E_t(x_t + x_{t+1}l_{t+1}^*) \left(\frac{1}{(1+\tau_{C,t+1})P_{t+1}} \right) \left\{ \frac{(C_{t+1}^s)^{\sigma-1} [(C_{t+1}^s)^\sigma + \kappa(ES_{t+1}^s)^\sigma]^{1/\sigma}}{[(C_{t+1}^s)^\sigma + \kappa(ES_{t+1}^s)^\sigma]^{1/\sigma} + \alpha_{CG}GC_{t+1}} \right\}$$

$$(18) \quad \left(\frac{1}{(1+\tau_{C,t})P_t} \right) \left\{ \frac{(C_t^s)^{\sigma-1} [(C_t^s)^\sigma + \kappa(ES_t^s)^\sigma]^{1/\sigma}}{[(C_t^s)^\sigma + \kappa(ES_t^s)^\sigma]^{1/\sigma} + \alpha_{CG}GC_t} \right\} = \beta E_t(1 + i_{t+1}) \left(\frac{1}{(1+\tau_{C,t+1})P_{t+1}} \right) \left\{ \frac{(C_{t+1}^s)^{\sigma-1} [(C_{t+1}^s)^\sigma + \kappa(ES_{t+1}^s)^\sigma]^{1/\sigma}}{[(C_{t+1}^s)^\sigma + \kappa(ES_{t+1}^s)^\sigma]^{1/\sigma} + \alpha_{CG}GC_{t+1}} \right\}$$

Finally, we have the conditions for the optimal selection of gross fixed investment and the optimal path of fixed capital stock in the tradable and non-tradable goods sectors:

$$(19) \quad q_{T,t} = P_t \left[1 + h_T \left(\frac{I_{T,t}}{K_{T,t-1}} - \delta_T \right) \right]$$

$$(20) \quad q_{NT,t} = P_t \left[1 + h_{NT} \left(\frac{I_{NT,t}}{K_{NT,t-1}} - \delta_{NT} \right) \right]$$

$$(21) \quad q_{T,t} = E_t \frac{1}{1 + \frac{x_{t+1}l_{t+1}^*}{x_t}} \left[q_{T,t+1}(1 - \delta_T) + (1 - \tau_{K,t+1})i_{T,t+1} - P_{t+1} \frac{h_T}{2} \left(\frac{I_{T,t+1}}{K_{T,t}} - \delta_T \right) \left(-\frac{I_{T,t+1}}{K_{T,t}} - \delta_T \right) \right]$$

$$(22) \quad q_{NT,t} = E_t \frac{1}{1 + \frac{x_{t+1}l_{t+1}^*}{x_t}} \left[q_{NT,t+1}(1 - \delta_{NT}) + (1 - \tau_{K,t+1})i_{NT,t+1} - P_{t+1} \frac{h_{NT}}{2} \left(\frac{I_{NT,t+1}}{K_{NT,t}} - \delta_{NT} \right) \left(-\frac{I_{NT,t+1}}{K_{NT,t}} - \delta_{NT} \right) \right]$$

where $q_{T,t} = \lambda_{T,t}/\lambda_{BC,t}$ and $q_{NT,t} = \lambda_{NT,t}/\lambda_{BC,t}$ (the λ 's are Lagrange multipliers associated with the budget and capital accumulation constraints) are Tobin's Q-like variables.

5.1.2 The non-Saudi household

Non-Saudis consume the final consumption good and energy services and supply labor to the tradable and non-tradable goods sectors. Their budget constraint includes only after-tax labor income for the hours of work supplied to the tradable and non-tradable goods sectors. Expenditure is on tradable, non-tradable consumption goods, and energy services. Non-Saudis transfer their savings abroad as remittances.

We assume the labor supply l^{ns} of the non-Saudi household to be exogenous. We denote the non-Saudi household by the subscript 'ns'. We also assume that non-Saudi workers do not work in the public sector. The non-Saudi household supplies labor to both the tradable and non-tradable goods sectors: see expressions (7)-(9) above.

As previously outlined, the only source of income of the non-Saudi household is labor. Thus, the non-Saudi's net income is

$$(23) \quad (1 - \tau_{w,t}^{ns})W_t^{ns}l_t^{ns}$$

We assume that the non-Saudi household spends a constant fraction of its income on consumption goods and energy services, and saves the rest of its income. The household transfers the savings as remittances. Letting the fractions be given by φ_c and φ_{es} we have

$$(24) \quad (1 + \tau_{c,t})P_t C_t^{ns} = \varphi_c (1 - \tau_{w,t}^{ns})W_t^{ns}l_t^{ns}$$

$$(25) \quad (1 + \tau_{es,t})P_{es,t}ES_t^{ns} = \varphi_{es} (1 - \tau_{w,t}^{ns})W_t^{ns}l_t^{ns}$$

$$(26) \quad (1 + \tau_{rem,t})REM_t = (1 - \varphi_c - \varphi_{es})(1 - \tau_{w,t}^{ns})W_t^{ns}l_t^{ns}$$

where, τ_{rem} is a tax on remittances. This specification implies that the non-Saudi household does not intertemporally optimize its decisions, and the sources of income and expenditures are exogenously determined. An important reason for this assumption is the lack of necessary data needed to calibrate the model.

5.2 Firms

There are several production activities in the economy, which can be classified according to the output produced. There are three types of final outputs: tradable and non-tradable goods, and energy services. There are two types of intermediate goods - energy services and electricity.

5.2.1 The tradable goods firm

We begin with the production activity of the tradable goods firm. These goods are produced using labor from both Saudi and non-Saudi workers, fixed capital and energy services. In keeping with the households' optimization problems, inputs and outputs are expressed in per capita terms (strictly speaking, per member of the labor force). We characterize the technology as the combination of aggregate labor input and a capital-energy services aggregate. Within the labor aggregate, two types of labor inputs are assumed to be perfectly substitutable, the capital-energy aggregate is a constant elasticity of substitution (CES) aggregator, and the two categories of inputs are combined in constant returns to scale Cobb-Douglas technology. The parametrized production function is the following:

$$(27) \quad y_{T,t} = z_{T,t}A_T[(1 - xpat)l_{T,t}^s + xpatl_{T,t}^{ns}]^{\alpha_T}\{\theta_T[(1 - xpat)K_{T,t-1}]^{-\nu_T} + (1 - \theta_T)ES_{T,t}^{-\nu_T}\}^{-\frac{(1-\alpha_T)}{\nu_T}}$$

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where, y_T is the aggregate output, l_T is the per capita amount of labor, K_T is the per capita stock of capital and ES_T is an aggregate amount of energy services in the tradable goods sector. $xpat$ is the share of non-Saudis in the total labor force, assumed to be constant in the current version of the model. α_T is the share of labor expenditure in total output. v_T is the elasticity of substitution between the capital stock and energy services in the capital-energy aggregate. θ_T is the share parameter of the stock of capital in the capital-energy aggregate. A_T is a technology progress indicator and $z_{T,t}$ is a shock to productivity in the tradable goods sector evolving as an autoregressive AR(1) process:

$$(28) \quad \ln z_{T,t} = \rho_T \ln z_{T,t-1} + \epsilon_{T,t} \quad \text{with} \quad \epsilon_{T,t} \sim N(0, \sigma_T^2)$$

where, ϵ_T is white noise, and ρ_T and σ_T are the persistency and standard deviation of the productivity shock. Noting that all firms in the model are assumed to be price-takers, the tradable goods firm maximizes profits by selecting the two types of labor, capital and energy services according to the following optimality rules:

$$(29) \quad W_{T,t}^s = \alpha_T P_{T,t} z_{T,t} A_T [(1 - xpat) l_{T,t}^s + xpat l_{T,t}^{ns}]^{\alpha_T - 1} \{ \theta_T [(1 - xpat) K_{T,t-1}]^{-v_T} + (1 - \theta_T) ES_{T,t}^{-v_T} \}^{\frac{-(1-\alpha_T)}{v_T}}$$

$$(30) \quad W_{T,t}^{ns} = \alpha_T P_{T,t} z_{T,t} A_T [(1 - xpat) l_{T,t}^s + xpat l_{T,t}^{ns}]^{\alpha_T - 1} \{ \theta_T [(1 - xpat) K_{T,t-1}]^{-v_T} + (1 - \theta_T) ES_{T,t}^{-v_T} \}^{\frac{-(1-\alpha_T)}{v_T}}$$

$$(31) \quad i_{T,t} = (1 - \alpha_T) \theta_T P_{T,t} z_{T,t} A_T [(1 - xpat) K_{T,t-1}]^{-v_T - 1} [(1 - xpat) l_{T,t}^s + xpat l_{T,t}^{ns}]^{\alpha_T} \{ \theta_T [(1 - xpat) K_{T,t-1}]^{-v_T} + (1 - \theta_T) ES_{T,t}^{-v_T} \}^{\frac{-(1-\alpha_T)-1}{v_T}}$$

$$(32) \quad P_{ES,t} = (1 - \alpha_T)(1 - \theta_T) P_{T,t} z_{T,t} A_T ES_{T,t}^{-v_T - 1} [(1 - xpat) l_{T,t}^s + xpat l_{T,t}^{ns}]^{\alpha_T} \{ \theta_T [(1 - xpat) K_{T,t-1}]^{-v_T} + (1 - \theta_T) ES_{T,t}^{-v_T} \}^{\frac{-(1-\alpha_T)-1}{v_T}}$$

Note that, as the two types of labor inputs are perfectly substitutable, the corresponding marginal productivities are equal, so from (29) and (30):

$$(33) \quad W_{T,t}^s = W_{T,t}^{ns}$$

so, Saudi and non-Saudi wages are equal in the sector. Note as well that, because the technology is characterized by constant returns to scale, by assumption we have:

$$(34) \quad \alpha_T P_{T,t} y_{T,t} = (1 - xpat) W_{T,t}^s l_{T,t}^s + xpat W_{T,t}^{ns} l_{T,t}^{ns}$$

$$(35) \quad (1 - \alpha_T) P_{T,t} y_{T,t} = (1 - xpat) i_{T,t} K_{T,t-1} + P_{ES,t} ES_{T,t}$$

so that profits are zero.

5.2.2 The non-tradable goods firm

Moving to the production and the optimal behavior of the non-tradable goods firm, we have expressions corresponding to the ones contained in the previous subsection. The production technology has the same structural representation:

$$(36) \quad y_{NT,t} = z_{NT,t} A_{NT} [(1 - xpat) l_{NT,t}^s + xpat l_{NT,t}^{ns}]^{\alpha_{NT}} \{ \theta_{NT} [(1 - xpat) K_{NT,t-1}]^{-v_{NT}} + (1 - \theta_{NT}) ES_{NT,t}^{-v_{NT}} \}^{\frac{-(1-\alpha_{NT})}{v_{NT}}}$$

where A_{NT} is a technology progress indicator and $z_{NT,t}$ is a shock to productivity which evolves as an AR(1) process:

$$(37) \quad \ln z_{NT,t} = \rho_{NT} \ln z_{NT,t-1} + \epsilon_{NT,t} \quad \text{with} \quad \epsilon_{NT,t} \sim N(0, \sigma_{NT}^2)$$

The profit-maximizing conditions for inputs are

$$(38) \quad W_{NT,t}^s = \alpha_{NT} P_{NT,t} z_{NT,t} A_{NT} [(1 - xpat) l_{NT,t}^s + xpat l_{NT,t}^{ns}]^{\alpha_{NT} - 1} \{ \theta_{NT} [(1 - xpat) K_{NT,t-1}]^{-v_{NT}} + (1 - \theta_{NT}) ES_{NT,t}^{-v_{NT}} \}^{\frac{-(1-\alpha_{NT})}{v_{NT}}}$$

$$(39) \quad W_{NT,t}^{ns} = \alpha_{NT} P_{NT,t} z_{NT,t} A_{NT} [(1 - xpat) l_{NT,t}^s + xpat l_{NT,t}^{ns}]^{\alpha_{NT} - 1} \{ \theta_{NT} [(1 - xpat) K_{NT,t-1}]^{-v_{NT}} + (1 - \theta_{NT}) ES_{NT,t}^{-v_{NT}} \}^{\frac{-(1-\alpha_{NT})}{v_{NT}}}$$

$$(40) \quad i_{NT,t} = (1 - \alpha_{NT})\theta_{NT}P_{NT,t}Z_{NT,t}A_{NT}[(1 - xpat)K_{NT,t-1}]^{-\nu_{NT}-1}[(1 - xpat)l_{NT,t}^s + xpatl_{NT,t}^{ns}]^{\alpha_{NT}}\{\theta_{NT}[(1 - xpat)K_{NT,t-1}]^{-\nu_{NT}} + (1 - \theta_{NT})ES_{NT,t}^{-\nu_{NT}}\}^{\frac{-(1-\alpha_{NT})-1}{\nu_{NT}}}$$

$$(41) \quad P_{ES,t} = (1 - \alpha_{NT})(1 - \theta_{NT})P_{NT,t}Z_{NT,t}A_{NT}ES_{NT,t}^{-\nu_{NT}-1}[(1 - xpat)l_{NT,t}^s + xpatl_{NT,t}^{ns}]^{\alpha_{NT}}\{\theta_{NT}[(1 - xpat)K_{NT,t-1}]^{-\nu_{NT}} + (1 - \theta_{NT})ES_{NT,t}^{-\nu_{NT}}\}^{\frac{-(1-\alpha_{NT})}{\nu_{NT}}-1}$$

Also in this case, (38) and (39) imply the equality of the marginal productivities and therefore of wage rates:

$$(42) \quad W_{NT,t}^s = W_{NT,t}^{ns}$$

In the present case, profits are also zero:

$$(43) \quad \alpha_{NT}P_{NT,t}y_{NT,t} = (1 - xpat)W_{NT,t}^s l_{NT,t}^s + xpatW_{NT,t}^{ns} l_{NT,t}^{ns}$$

$$(44) \quad (1 - \alpha_{NT})P_{NT,t}y_{NT,t} = (1 - xpat)i_{NT,t}K_{NT,t-1} + P_{ES,t}ES_{NT,t}$$

5.2.3 The energy services firm

This firm is a price-taker and produces energy-related services such as air conditioning, use of electrical appliances, lighting, and private transportation. For simplicity, we assume that energy services are provided using energy sources as the only inputs. Specifically, electricity (EL_{ES}), oil (O_{ES}) and gas (G_{ES}) are combined according to a CES technology with an elasticity of input substitution equal to $1/(1-\lambda)$. Specifically:

$$(45) \quad ES_t = [\varphi EL_t^\lambda + \psi O_{ES,t}^\lambda + (1 - \varphi - \psi)G_{ES,t}^\lambda]^{1/\lambda}$$

where, φ and ψ are the share parameters of electricity and oil in the CES technology. Maximizing profits with respect to the optimal choice of inputs yields the following optimality conditions:

$$(46) \quad P_{ES,t} = \left(\frac{EL_t^{1-\lambda}}{\varphi}\right) [\varphi EL_t^\lambda + \psi O_{ES,t}^\lambda + (1 - \varphi - \psi)G_{ES,t}^\lambda]^{\frac{\lambda-1}{\lambda}} P_{EL,t}$$

$$(47) \quad P_{ES,t} = \left(\frac{O_{ES,t}^{1-\lambda}}{\psi}\right) [\varphi EL_t^\lambda + \psi O_{ES,t}^\lambda + (1 - \varphi - \psi)G_{ES,t}^\lambda]^{\frac{\lambda-1}{\lambda}} P_{O,t}^d$$

$$(48) \quad P_{ES,t} = \left(\frac{G_{ES,t}^{1-\lambda}}{1-\varphi-\psi}\right) [\varphi EL_t^\lambda + \psi O_{ES,t}^\lambda + (1 - \varphi - \psi)G_{ES,t}^\lambda]^{\frac{\lambda-1}{\lambda}} P_{G,t}^d$$

where, P_O^d and P_G^d are the domestic prices of oil and gas; we assume these prices are fixed. Note that the Saudi government administers these prices and these administered prices are lower than their international counterparts. The constant returns to scale and profit-maximizing behavior jointly imply zero profits for the energy services firm.

5.2.4 The electricity generation firm

Electricity is generated using three fuels: oil, gas and a renewable source such as solar. Oil and gas inputs admit a degree of substitution, modeled as a Cobb-Douglas fossil fuels aggregate (with a unitary elasticity of substitution between oil and gas). This fossil fuels aggregate input and renewables (REN) are then assumed to be perfect substitutes. Returns to scale are assumed to be constant. Specifically:

$$(49) \quad EL_t = O_{EL,t}^\gamma G_{EL,t}^{1-\gamma} + REN_t$$

where, γ is the share of oil expenditure in total electricity output and REN is the level of renewable energy. Maximizing profits with respect to the optimal choice of inputs yields the following optimality conditions:

$$(50) \quad \gamma P_{EL,t} O_{EL,t}^{\gamma-1} G_{EL,t}^{1-\gamma} = P_{O,t}^d$$

$$(51) \quad (1 - \gamma) P_{EL,t} O_{EL,t}^\gamma G_{EL,t}^{-\gamma} = P_{G,t}^d$$

so that the optimal combination of the two fossil fuels is governed by the familiar rule:

$$(52) \quad \frac{\gamma}{1-\gamma} \frac{G_{EL,t}}{O_{EL,t}} = \frac{P_{G,t}^d}{P_{O,t}^d}$$

The condition for renewables is:

$$(53) \quad P_{EL,t} = P_{R,t}$$

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where, P_R is the price of the renewable energy source. Thus, the price of renewable energy is equal to the electricity price. If we substitute (49)-(51) and (53) into the definition of profits, we can see that profits in this case are also zero.

5.3 Natural resources

Natural resources are represented by renewable energy, oil and gas. We assume that they are produced using a simple technology that depends only on the stock of public capital.

First, we assume that renewable energy is produced using public capital, which implies that the government, via the state-owned electricity company, invests directly in this technology. Renewable energy is produced according to:

$$(54) \quad REN_t = \left[\frac{A_R}{1 + \chi \left(\frac{REN_t}{EL_t} \right)} \right] K_{R,t-1}^g$$

where, A_R is the productivity of the public capital stock (K_R^g) in renewable production and χ is the integration cost parameter. The term in square brackets represents the productivity of a unit of public capital invested in renewables or, conversely, the cost of one unit of renewable energy. Note that the productivity of renewables depends negatively on renewable penetration in electricity generation, reflecting integration costs to the electric system. Public capital accumulation in renewables is given by:

$$(55) \quad K_{R,t}^g = I_{R,t}^g + (1 - \delta_R) K_{R,t-1}^g$$

where, I_R^g is public investment in renewable energy and δ_R is the depreciation rate of the capital stock in renewables. Turning to oil we have:

$$(56) \quad \bar{O}_t = \alpha_O u_t K_{O,t-1}^g$$

where, α_O is the productivity of public capital stock (K_O^g) in oil production, u is the rate of capacity utilization of capital allocated to oil and $\bar{O}$ is total oil production. Public capital in oil production is given by:

$$(57) \quad K_{O,t}^g = I_{O,t}^g + (1 - \delta_O) K_{O,t-1}^g$$

Next, we assume that gas production (G) is proportional to oil production:

$$(58) \quad \bar{G}_t = \alpha_G \bar{O}_t$$

where, α_G is the proportionality factor of oil. Note that gas production also depends on public capital investment through (56).

The international price of oil is assumed to evolve over time according to a mean-reverting process. Since Saudi Arabia has a significant market share (about 12%) of the global oil market, any changes to Saudi oil exports will have an impact on international prices and can significantly impact Saudi export revenues. Thus, we need to model the price changes caused by variations in Saudi exports. We consider a mean reverting process that embeds two adjustment mechanisms: a) a partial adjustment toward the price that would materialize in the long-run if the previous period's level of exports remained forever, and b) a short-run adjustment due to the current change in export levels, as given below:

$$(59) \quad \ln P_{O,t}^* = (1 - \varrho_O) \ln P_{O,ss,t-1}^* + \varrho_O \ln P_{O,t-1}^* + \frac{1}{\varepsilon_{SR}} \ln \frac{OX_t}{OX_{t-1}} + \epsilon_{O,t}$$

$$(60) \quad \ln \frac{P_{O,ss,t}^*}{P_{O,ss}^*} = \frac{1}{\varepsilon_{LR}} \ln \frac{OX_t}{OX_{ss}}$$

where, $P_{O,t}^*$ is the international price of oil in each period, ϱ_O is the persistency of the international price, $P_{O,ss,t}^*$ is the international price of oil that would be observed in the steady-state if Saudi oil exports

were at the level of OX_t in the long-run. In our model, ε_{SR} and ε_{LR} are, respectively, the short-run and the long-run price elasticities of residual demand for Saudi oil exports. The term $\epsilon_{0,t}$ is a white noise innovation, $\epsilon_{0,t} \sim N(0, \sigma_0^2)$. Mathematically, it can be proved that $\varepsilon_{SR} = \frac{\varepsilon_{SR}^D - \varepsilon_{SR}^S(1-\rho)}{\rho}$ and $\varepsilon_{LR} = \frac{\varepsilon_{LR}^D - \varepsilon_{LR}^S(1-\rho)}{\rho}$, where ρ is the market share of Saudi oil exports in the global oil market, and ε_{SR}^D , ε_{SR}^S are, respectively, the price elasticity of global oil demand and price elasticity of non-Saudi oil supply, in the short-run, while ε_{LR}^D and ε_{LR}^S , represent the same elasticities in the long-run.

Given the definition of the short-term elasticity ε_{SR} , the term $\frac{1}{\varepsilon_{SR}} \ln \frac{OX_t}{OX_{t-1}}$ represents the change in the oil price due to the variation in export levels observed during the period. Equation (60) captures the long-run response of the international oil price to changes in Saudi oil export levels. The combination of (59) and (60) captures both the short-term effects due to contemporaneous events and the long-run effects due to past changes in export levels.

If the short-term price elasticity ε_{SR} and the long-term price elasticity ε_{LR} of residual demand for Saudi oil were both infinite, we return to the standard assumption of a competitive market, and (59) would simplify to the usual mean-reverting equation with a constant long-run price as shown below:

$$\ln P_{0,t}^* = (1 - \varrho_0) \ln P_{0,ss}^* + \varrho_0 \ln P_{0,t-1}^* + \epsilon_{0,t}$$

The international price of gas is simply assumed to follow an AR(1) process as follows:

$$(61) \quad \ln P_{G,t}^* = (1 - \varrho_G) \ln P_{G,ss}^* + \varrho_G \ln P_{G,t-1}^* + \epsilon_{G,t}$$

Where, ϱ_G is the persistency of the international gas price, $P_{G,ss}^*$ is the steady-state international price of gas and the innovation is white noise, $\epsilon_{G,t} \sim N(0, \sigma_G^2)$.

5.4 Government

The government budget equates revenues to outlays. Expenditures include transfers, public investment in renewable energy and oil and gas production, and labor compensation to public employees. On the revenue side, we have tax payments on the consumption of goods and energy services, labor taxes paid by both Saudi and non-Saudi households, capital income taxes paid by Saudi households, and taxes on remittances paid by non-Saudi households. In addition, there are revenues from domestic sales of oil and gas and exports and net issues of government bonds. The government budget is:

$$(62) \quad \begin{aligned} & \tau_{C,t} P_t [(1 - xpat) C_t^S + (xpat) C_t^{ns}] + \tau_{ES,t} P_{ES,t} [(1 - xpat) ES_t^S + (xpat) ES_t^{ns}] + \tau_{W,t}^S [W_t^S l_t^S + W_{g,t}^S l_{g,t}^S] (1 - xpat) + \tau_{W,t}^{ns} W_t^{ns} l_t^{ns} (xpat) + \\ & \tau_{REM,t} REM_t (xpat) + \tau_{K,t} [i_{T,t} K_{T,t-1} + i_{NT,t} K_{NT,t-1}] (1 - xpat) + P_{0,t}^d (O_{EL,t} + O_{ES,t}) + P_{G,t}^d (G_{EL,t} + G_{ES,t}) + P_{R,t} REN_t + x_t P_{0,t}^* (\bar{O}_t - O_{EL,t} - O_{ES,t}) + \\ & x_t P_{G,t}^* (\bar{G}_t - G_{EL,t} - G_{ES,t}) + [B_t - (1 + i_t) B_{t-1}] (1 - xpat) = P_t l_{0,t}^g + P_t l_{R,t}^g + \\ & W_{g,t}^S l_{g,t}^S (1 - xpat) + P_t GC_t (1 - xpat) + TR_t (1 - xpat) \end{aligned}$$

The fiscal authority controls a number of policy variables that can be used to pursue its own goals, such as tax rates, government consumption and public investments. These variables are exogenous to the model and are listed in Appendix III. We assume that transfers keep the budget balanced in each period.

5.5 Foreign sector

The foreign sector is characterized by a current account with imported goods (a component of tradable goods), with remittances on one side and exports of tradable goods, oil and gas on the other. Net interest payments on foreign debt are included in the current account. The capital account includes net acquisitions of foreign debt and the change in official holdings (official reserves) which ensure an equilibrium of the balance of payments.

5.6 Market equilibrium, aggregation and foreign asset position

The model description is completed by the equilibrium conditions of the markets of this model economy and by the aggregate resource constraint.

Firstly, achieving equilibrium in the market for energy services requires equating the production of energy services with the demand by Saudi and non-Saudi households and by firms producing tradable and non-tradable consumption goods. That is:

$$(63) \quad ES_t = (1 - xpat)ES_t^S + (xpat)ES_t^{NS} + ES_{T,t} + ES_{NT,t}$$

Secondly, note that the output of electricity generation is only used as an input to the energy services firm. This means that (45) and (49) jointly imply that the equilibrium for electricity is satisfied.

Invoking Walras' law, we finally impose the market equilibrium in the non-tradable goods sectors. The supply of non-tradable goods is specified in (36). This is equated to the demand for these goods by Saudi and non-Saudi households for consumption as well as for investment in fixed capital and government services. The market equilibrium condition is:

$$(64) \quad y_{NT,t} = (1 - \theta_C) \left[\frac{(1 + \tau_{NT,t})P_{NT,t}}{(1 + \tau_{C,t})P_t} \right]^{-\sigma_C} [(1 - xpat)C_t^S + (xpat)C_t^{NS}] + (1 - \theta_C) \left(\frac{P_{NT,t}}{P_t} \right)^{-\sigma_C} \left\{ (1 - xpat) \left[I_{T,t} + \frac{h_T}{2} \left(\frac{I_{T,t}}{K_{T,t-1}} - \delta_T \right)^2 K_{T,t-1} + I_{NT,t} + \frac{h_{NT}}{2} \left(\frac{I_{NT,t}}{K_{NT,t-1}} - \delta_{NT} \right)^2 K_{NT,t-1} \right] + (I_{O,t}^g + I_{R,t}^g) + (1 - xpat)GC_t \right\}$$

To close the model, we have an economy-wide constraint that ensures that all resources used by the economic agents are equal to all resources created or available. This entails using the household budget constraint for both Saudi and non-Saudi agents and the government budget constraint, together with households' optimality

conditions and firms' behaviors. This shows that the aggregate resource constraint is satisfied in this model economy.

For ease of exposition, and to limit the clutter of notation, we assume that the tax rates on tradable and non-tradable goods consumption are equal, i.e., $\tau_{T,t} = \tau_{NT,t}$. Of course, this assumption can be easily relaxed. In addition, we denote the expressions for capital adjustment costs by $AC_{T,t}$ and $AC_{NT,t}$. After rearrangements (fully described in Appendix II) we obtain the final version of the constraint as follows:

$$(65) \quad P_t[(1 - xpat)C_t^S + (xpat)C_t^{NS}] + P_{ES,t}[(1 - xpat)ES_t^S + (xpat)ES_t^{NS}] + (1 - xpat)P_t[(I_{T,t} + AC_{T,t}) + (I_{NT,t} + AC_{NT,t})] + P_t(I_{O,t}^g + I_{R,t}^g) + (1 - xpat)P_tGC_t + (1 - xpat)[x_t B_t^* - (x_{t-1} + x_t i_t^*)B_{t-1}^*] + (xpat)REM_t = P_{T,t}y_{T,t} + P_{NT,t}y_{NT,t} + P_{ES,t}[(1 - xpat)ES_t^S + (xpat)ES_t^{NS}] + x_t P_{O,t}(\bar{O}_t - O_{EL,t} - O_{ES,t}) + x_t P_{G,t}(\bar{G}_t - G_{EL,t} - G_{ES,t})$$

Expression (65) is the basic accounting identity of this model economy. Indeed, note that on the left-hand side:

- the term $\{P_t[(1 - xpat)C_t^S + (xpat)C_t^{NS}] + P_{ES,t}[(1 - xpat)ES_t^S + (xpat)ES_t^{NS}]\}$ is private final consumption;
- the term $\{(1 - xpat)P_t[(I_{T,t} + AC_{T,t}) + (I_{NT,t} + AC_{NT,t})]\}$ is private investment;
- the term $\{P_t(I_{O,t}^g + I_{R,t}^g)\}$ is public investment;
- the term $\{(1 - xpat)P_tGC_t\}$ is public consumption;
- the term $\{(1 - xpat)[x_t B_t^* - (x_{t-1} + x_t i_t^*)B_{t-1}^*] + (xpat)REM_t\}$ is the current account.

On the right-hand side:

- the term $\{P_{T,t}y_{T,t} + P_{NT,t}y_{NT,t} + P_{ES,t}[(1 - xpat)ES_t^S + (xpat)ES_t^{NS}] + x_t P_{O,t}(\bar{O}_t - O_{EL,t} - O_{ES,t}) + x_t P_{G,t}(\bar{G}_t - G_{EL,t} - G_{ES,t})\}$ is total nominal value added or nominal GDP.

Thus, nominal GDP at current prices for this economy is given by:

$$(66) \quad GDP_t = P_{T,t}Y_{T,t} + P_{NT,t}Y_{NT,t} + P_{ES,t}[(1 - xpat)ES_t^S + (xpat)ES_t^{NS}] + x_t P_{O,t}(\bar{O}_t - O_{EL,t} - O_{ES,t}) + x_t P_{G,t}(\bar{G}_t - G_{EL,t} - G_{ES,t})$$

In the model, the trade balance, in nominal terms, is the sum of remittances by expats, the change in the foreign bond position and interest earned from foreign bonds. That is:

$$(67) \quad TB_t = (1 - xpat)[x_t B_t^* - (x_{t-1} + x_t i_t^*)B_{t-1}^*] + (xpat)REM_t$$

To obtain the real GDP and the real trade balance we divide (66) and (67) by the aggregate price (P) of tradable and non-tradable goods obtained from Equation (6).

To close the model, we need to select a normalization rule, here assumed to be the following:

$$(68) \quad P_{T,t} = x_t P_{T,t}^*$$

where, $P_{T,t}^*$ is the international price of the tradable good and is exogenous.

5.7 CO₂ emissions

The model includes the calculation of carbon dioxide (CO₂) emissions generated by the combustion of oil and gas in electricity generation and the production of energy services. Since we do not explicitly model other production activities such as agriculture, the emissions of CO₂ considered by the model only accounts for greenhouse gas emissions that are generated by fossil fuels. We therefore have:

$$(69) \quad CO_2 = ef_O(O_{ES} + O_{EL}) + ef_G(G_{ES} + G_{EL})$$

where, CO₂ is carbon dioxide emissions and ef_O and ef_G are emission factors for oil and gas use, respectively.

6. Parametrization and model solution

The model is calibrated on an annual basis, as are most of the data for Saudi Arabia in this frequency. We have three types of parameters and exogenous variables depending on the calibration procedure: (i) parameters obtained from the literature, (ii) parameters independently calibrated and (iii) parameters jointly calibrated for the model to match some of the long-run relationships of the Saudi economy from 1987-2016. The first two kinds of parameters are presented in Table 1, while parameters jointly calibrated are reported in Table 2, and the ratios to be matched are presented in Table 3.

6.1 Prices, interest rate

As is usual in models with a tradable and a non-tradable sector, tradable goods are set as the numeraire, so we assume $P_{T,t} = 1 \forall t$. Following Blazquez, Hunt and Manzano (2017), the risk-free international interest rate (i^{**}) is set to 4%, while ζ is set to reproduce the volatility of the trade balance over the GDP observed in the data.

6.2 Saudi household

We assume $\sigma_c = 1$, implying a Cobb-Douglas aggregator for tradable and non-tradable goods. The weights of tradables (θ_c), energy services (κ) and public services (α_{GC}), and the size of the government services (GC), are jointly calibrated with other parameters to match the share of energy services expenditure over the total consumption expenditure, the share of private consumption over GDP and the share of public consumption over GDP. The elasticity of substitution between aggregate consumption and energy services (σ) is set to -0.3333, following Plante (2014).

The inverse of the Frisch labor supply elasticity is set at $\nu = 1.429$ to represent the long-run labor supply elasticity (see Behar [2015]). The weight of disutility from the labor supply (η) is jointly calibrated to

match the ratio of Saudi labor to the total labor supply in the long-run. The discount factor (β) is obtained from the first order condition for foreign bonds in steady state. The size of the foreign bonds (B^*) and government bonds (B) in the initial steady state are jointly calibrated to reproduce the ratios of the trade balance and government bonds over GDP.

The elasticity of substitution between the tradable and non-tradable labor supply is set at $\sigma_L = -1$ (see Berg et al. [2010]), while the share of labor in the tradable sector (θ_L^s) is jointly calibrated to match the average ratio of labor in the tradable sector to total labor.

6.3 Non-Saudi households

The data on non-Saudis in the total labor force (x_{pat}) is taken from the General Authority of Statistics (2016). We assume the same goods and labor aggregators as used for Saudi households, with the same parameter values. The parameters that define the fraction of income spent on consumer goods (φ_C) and energy services (φ_{ES}), are jointly calibrated to match the share of energy services expenditure over total expenditure and the ratio of remittances to GDP.

6.4 Tradable and non-tradable goods sectors

We assume most of the production function parameters are the same for the tradable and the non-tradable sectors. The main reason for this assumption is the lack of data at the sectoral level. We set $\alpha_T = \alpha_{NT} = 0.58$, following Blazquez, Hunt and Manzano (2017), and the elasticity of substitution between capital and energy services $v_T = v_{NT} = -1.381$, from Koetse, de Groot, and Florax (2008).

To determine the share of capital in the capital-energy services aggregate, (θ_T, θ_{NT}) , we build the capital stock by using data for gross fixed capital formation. We start by imposing a long-run capital-

to-output ratio (k/y) equal to 2 and assume a 10% annual depreciation rate of capital, which is a common assumption in the macroeconomic literature (King and Rebelo 1999). The capital-output ratio is consistent with 20% gross fixed capital formation over GDP, the average value between 2000 and 2017. We then estimate capital stock levels in 2017 using the ratio $K_{2017}/GDP_{2017} = 1.75$. Using the optimality condition, according to which the relative demand for capital stock to energy services is a function of their price ratio, we obtain $\theta_T = \theta_{NT} = 0.99997$. Following Koetse, de Groot, and Florax (2008), the parameter governing the elasticity of substitution between capital stock and energy services is set at 1.381 for both sectors. The parameters for the adjustment cost of capital (h_T, h_{NT}), are calibrated to match the volatility of investment relative to GDP. The technology index for tradables and non-tradables (A_T, A_{NT}) are jointly calibrated to reproduce the relative weight of the two sectors.

We assume that the total factor productivity shocks in the tradable and non-tradable sectors follow AR(1) processes. Since these shocks may not be independent, these AR(1) processes given by (28) and (37) along with the stochastic processes for the international prices of oil and gas are given by (59, 60) and (61) have been estimated using the seemingly unrelated regression (SUR) method. These total factor productivities in both sectors in the steady-state are normalized to 1. Our estimates show that the productivity shocks in the tradable sector is relatively persistent and more volatile than the productivity shock faced by the non-tradable sector during the period 1997-2016. The persistency parameters of the shocks in the tradable and non-tradable sectors are 0.53 and 0.45 respectively. The standard deviations of the shocks in the sectors are 0.022 and 0.036 respectively.

6.5 Energy services and electricity

We obtain the fuel consumption in power generation from the BP Statistical Review of World Energy (2016) and the Electricity and Cogeneration Regulation Authority (2017), and the price of electricity from the Saudi Electricity Company (2018). Prices for gas and oil supply are obtained from Wogan, Pradhan, and Albardi (2017). Using these data for electricity generation, we set the share of oil $\gamma = 0.47$. In energy service production, we follow Blazquez, Hunt and Manzano (2017) to set the share of electricity $\varphi = 0.332$, while we assume $\psi = 0.332$, implying approximately the same weight for oil and gas in the production of energy services. The parameter governing the elasticity of substitution among oil, gas and electricity is set at $\lambda = -2.574$ (Serletis, Timilsina, and Vasetsky 2010).

6.6 Production of renewables

Following, Blazquez, Hunt, and Manzano (2017), we take the cost of public capital invested per MW in a representative photovoltaic technology for renewable power generation as 1.34 million SAR in 2010 prices. We also take the annual capacity factor to be 19% and the depreciation rate to be 5%, standard for these types of technologies. Using these data, we estimate that the marginal productivity of public capital in renewable energy technology is equal to 0.0328 toe per thousand SAR in 2010 prices. Thus, we set $A = 0.0328$. Mai et al. (2012) find that in the U.S. the cost of the electricity system increases by as much as 5% when renewables penetrate the system by 10%. Thus, we set the renewables' integration cost parameter $\chi = 0.5$. The depreciation of public capital in renewables (μ_R) is assumed to be 5%, as in Blazquez, Hunt and Manzano (2017).

6.7 Production of oil and gas

The marginal product of public capital in oil production (α_o) is jointly calibrated to match the ratio

of oil over oil and gas used in electricity generation. The depreciation of public capital in oil production (δ_o) is assumed to be 0.05. The exogenous value of public investment in the oil sector (I_O^g) is set to match the average oil production of Saudi Arabia. We normalize the rate of capital utilization (u) to 1. In the model we assume gas production is proportionate to the production of oil, and calibrate α_g accordingly.

6.8 International and domestic prices of oil and gas

The average price of Brent crude during 1997-2016 was found to be $P_{0,ss}^* = 1.753$ thousand SAR per toe (which corresponds to US\$ 65 per barrel in 2016, at constant 2010 prices). The average Saudi oil exports during the period was found to be $OX_{ss} = 18.06$ billion toe SAMA 2017. Following Pierru, Smith, and Zamrik (2018), we set the market share of Saudi oil exports in global oil output to $\rho = 0.12$. To obtain the long-run price elasticity of residual Saudi oil demand we assumed the long-run price elasticity of global oil demand to be about $\varepsilon_{LR}^D = -0.6$ and the supply of non-Saudi oil to be about $\varepsilon_{LR}^S = 0.3$. The long-run price elasticity of Saudi oil exports was therefore obtained to be equal to $\varepsilon_{LR} = -7.2$. To obtain the short-run price elasticity of Saudi oil exports, we assumed that these elasticity parameters in the short-run would be about 50% of their counterparts in the long-run. Hence, the short-run price elasticity of residual Saudi oil demand was $\varepsilon_{SR} = -3.6$. For the long-run price partial adjustment effect we estimate Equations (59, 60) for the period 1997-2016. As shocks to oil and gas prices might not be independent, the stochastic processes given by (59, 60) and (61) along with (28) and (37) have been estimated using the SUR method. The estimated coefficient was $\varrho_o = 0.97$ and the standard deviation of the innovation $\sigma_o = 0.2$.

The average international price of gas from 1997-2016 was found to be $P_{G,ss}^* = 0.669$ thousand SAR

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per toe (corresponding to US\$ 25 per barrel in 2016 at constant 2010 prices). The persistency parameter of the gas prices process during the period was $\rho_G = 0.6$ and the standard deviation of the innovation $\sigma_G = 0.157$.

The domestic prices of oil and gas (P_o^d, P_g^d) are jointly calibrated to match the observed ratios of oil and gas exports over production.

6.9 CO₂ emissions

We obtain historical data on CO₂ emissions for Saudi Arabia from the BP Statistical Review of World Energy 2016. CO₂ emissions in Saudi Arabia predominantly come from oil and gas combustion. For example, in 2014, fuel combustion in electricity generation accounted for over 48% of CO₂ emissions, followed by transport (24%) and industries (24%) (World Resources Institute 2015). Other activities, including fugitive emissions, account for about 3%. Our model does not explicitly account for other activities. Instead, we model CO₂ emissions using the Intergovernmental Panel on Climate Change (IPCC) default emissions factors ef_o

$= 3.07$ tonnes/toe for oil and $ef_g = 2.35$ tonnes/toe for gas use (IPCC 1996).

6.10 Calibration procedure

We broadly categorize the parameters and exogenous variables of the model by three types, according to the calibration procedure. As discussed earlier, some of the parameters are jointly calibrated to reproduce some of the long-run characteristics of the Saudi economy. The objective of the process is to match the model's steady-state results to key empirical relationships. We follow a non-linear constrained optimization procedure that minimizes the deviation from the empirical relationships while meeting the model's system of equations as constraints. In other words, the calibration procedure involves minimizing the sum of squares of the deviations from the empirical relationships by optimally adjusting the selected parameters. The jointly calibrated parameters are shown in Table 1, while the ratios of the Saudi economy and the corresponding ratios from the model solution are shown in Table 2. Table 3 presents the list of remaining parameter values and their references.

Table 1. Parameters from the literature or individually calibrated.

List of parameters			
Price normalization			
$P_{T,t}$	Price of tradable goods	1	Standard assumption
International interest rate			
i^{**}	Risk-free international interest rate	0.04	Blazquez, Hunt, and Manzano (2017)
ζ	Risk premium parameter	0.01	Individual calibration

Labor force

x_{pat}	Share of non-Saudi labor force	0.6	General Authority for Statistics (2016)
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Saudi household

σ_L	Elasticity of substitution between tradable and non-tradable labor supply	-1	Berg et al. (2010)
σ_c	Elasticity of substitution between tradable and non-tradable consumption goods	1	Assumption
β	Discount factor	0.9615	Author's calculation
σ	Elasticity of substitution between consumption and energy services	-0.3333	Plante (2014)
ν	Inverse of Frisch labor supply elasticity	1.429	Behar (2015)

Tradable and non-tradable firms

h_T	Capital adjustment cost tradable sector	1.2	Individual calibration
h_{NT}	Capital adjustment cost non-tradable sector	0.02	Individual calibration
δ_T, δ_{NT}	Capital depreciation rate	0.10	King and Rebelo (1999)
θ_T, θ_{NT}	Capital in share in the capital-energy services aggregate	0.9997	Individual calibration
α_T, α_{NT}	Output elasticity of labor	0.582	Blazquez, Hunt, and Manzano (2017)
ν_T, ν_{NT}	Elasticity of substitution between capital and energy services	1.381	Koetse, Groot, and Florax (2008)
ϱ_T	Persistence of tradable goods productivity shock process	0.53	Authors' estimation
σ_T	Standard deviation of innovation to tradable goods productivity shock	0.022	Authors' estimation
ϱ_{NT}	Standard deviation of innovation to non-tradable goods productivity shock	0.45	Authors' estimation
σ_{NT}	Standard deviation of innovation to non-tradable goods productivity shock	0.036	Authors' estimation

Energy services firm

λ	Elasticity of substitution parameter between electricity, oil and gas	-0.2579	Serletis, Timilsina, and Vasetsky (2010)
φ	Share of electricity	0.332	Blazquez, Hunt, and Manzano (2017)
ψ	Share of oil	0.332	Authors' assumption

Electricity generation firm

γ	Share of oil in fossil fuels aggregate	0.47	Authors' calculation
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Natural resources: renewables

A_R	Productivity of public capital stock in renewables production	0.0328	Blazquez, Hunt, and Manzano (2017)
χ	Integration cost parameter	0.5	
δ_R	Depreciation rate of renewable sector capital stock	0.05	

Natural resources: oil

u	Rate of capacity utilization of public capital	1	Assumption
δ_O	Depreciation rate of oil sector capital stock	0.05	Assumption
I_O^g	Public investment in the oil sector	0.179	Individual calibration
ϱ_O	Persistency of oil price process	0.97	Authors' estimation
$P_{O,ss}^*$	Long-run oil price in foreign currency	1.753	Authors' estimation
σ_O^2	Variance of innovation to oil price process	0.039	Authors' estimation
ρ	Market share of Saudi oil in global oil output	0.12	Pierru, Smith, and Zamrik (2018)
ε_{LR}^D	Long-run price elasticity of global oil demand	-0.6	Assumption
ε_{LR}^S	Long-run price elasticity of supply of non-Saudi oil	0.3	Assumption
ε_{LR}	Long-run price elasticity of Saudi oil exports	-7.2	Authors' calculation
ε_{SR}	Short-run price elasticity of Saudi oil exports	-3.6	Authors' calculation

Natural resources: gas

α_G	Proportionality factor between oil and gas production	0.185	Author calculation
ϱ_G	Persistency of gas price process	0.6	Authors' estimation
$P_{G,ss}^*$	Long-run natural gas price in foreign currency	0.669	Authors' estimation
σ_G^2	Variance of innovation to gas price process	0.025	Authors' estimation

Carbon dioxide emissions

ef_O	Emission factor for oil combustion	3.07	IPCC (1996)
ef_G	Emission factor for gas combustion	2.35	

Source: KAPSARC.

Table 2. Parameters and exogenous variables jointly calibrated.

Symbol	Description	Values
θ_c	Share of tradable goods in private consumption	0.479
κ	Weight of energy services in household consumption	0.0598
α_{GC}	Weight of public services in household consumption	0.452
GC	Public services in initial steady state	0.647
$\theta_L^s, \theta_L^{ns}$	Weight of the tradable in the labor aggregator	0.736
AT	Technology index in tradable goods production	0.434
ANT	Technology index in non-tradable goods production	0.577
P_O^d	Domestic price of oil	0.292
P_G^d	Domestic price of gas	0.409
α_O	Productivity of capital in oil production	0.128
B_{ss}	Government bonds	0.466
B_{ss}^*	Foreign bonds	-10.041
φ_c	Non-Saudi share of income spent on consumption goods	0.65
φ_{ES}	Non-Saudi share of income spent on energy services	0.10
η	Weight of labor disutility	0.429

Source: KAPSARC.

Table 3. Comparison of long-run relationships from historical data and calibrated model economy.

Long-run relationship (in ratios)	Historical data (1997-2016)	Calibrated model economy
Saudi labor to total labor supply	0.38	0.38
Share of tradable goods in total outputs	0.22	0.22
Oil exports to oil production	0.76	0.76
Gas exports to gas production	0	0
Trade-balance to nominal GDP	0.16	0.16
Private consumption to GDP	0.31	0.31

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Public consumption to GDP	0.22	0.22
Public bonds to GDP	0.13	0.13
Energy services expenditure over total consumption expenditure for non-Saudis	0.11	0.13
Remittances to GDP	0.05	0.05
Labor in tradable goods over total supply of labor	0.47	0.47
Energy services expenditure over total consumption expenditure for Saudis	0.11	0.11
Energy services expenditure over total consumption expenditure for non-Saudis	0.10	0.13
Share of oil in electricity demand for oil and gas	0.51	0.55

Source: KAPSARC.

7. Policy experiments

The present model is characterized by a fair degree of complexity for a small open economy DSGE. In its current version, it nevertheless lends itself to be used to conduct a set of interesting policy experiments on individual policy changes or the effect of joint policy decisions. We can also study the effects of price or technology shocks on the main variables of the model and the changes a policy decision can produce on the economy, and trace the dynamic adjustment from the initial to the final equilibrium of the economic system. These are the main virtues of DSGE models.

A list of potential policy changes includes the introduction of a value-added tax, investment in renewables deployment, a tax on expatriate income, an increase in the administered energy prices, taxation of Saudi labor and/or capital income. It is also possible to study policy changes, including more than one intervention.

8. Concluding remarks and future extensions

The DSGE model described above only contains real frictions, namely the costly adjustment of

fixed capital stocks. The present version does not, therefore, include features of New Keynesian models. Accounting for market imperfections and nominal frictions will be the first area of future development.

While the model contains a fairly detailed description of the role of energy in the economy, it does not currently model carbon emissions and their feedback on the economy. Therefore the effects of mitigation policies cannot be studied. This is the next step of model development in our research agenda.

Finally, in the present version money is not explicitly modeled and thus the model does not incorporate any role for active monetary policy. This is the third development that we aim to pursue.

This paper has presented what appears to be the first dynamic stochastic general equilibrium model for Saudi Arabia, a resource-rich small open economy. A DSGE model is characterized by one extremely important advantage: it enables an analysis of the impacts of shocks and policies as well as the assessment of policy changes and experiments in a ‘what-if’ conceptual framework.

References

- Alyousef, Yousef, and Paul Stevens. 2011. "The Cost of Domestic Energy Prices to Saudi Arabia." *Energy Policy* 39 (11). Elsevier: 6900–6905.
- Annicchiarico, Barbara, and Fabio Di Dio. 2015. "Environmental Policy and Macroeconomic Dynamics in a New Keynesian Model." *Journal of Environmental Economics and Management* 69 (January). Academic Press: 1–21.
- . 2017. "GHG Emissions Control and Monetary Policy." *Environmental and Resource Economics* 67 (4): 823–51.
- Behar, Alberto. 2015. "Comparing the Employment-Output Elasticities of Expatriates and Nationals in the Gulf Cooperation Council." WP/15/191. International Monetary Fund.
- Berg, Andrew, Jan Gottschalk, Rafael Portillo, and Luis-Felipe Zanna. 2010. "The Macroeconomics of Medium-Term Aid Scaling-up Scenarios." WP/10/160. International Monetary Fund.
- Blazquez, Jorge, Lester C. Hunt, Baltasar Manzano, and Axel Pierru. 2018. "The Value of Saving Oil in Saudi Arabia." KS-2018-DP030. KAPSARC Discussion Paper.
- Blazquez, Jorge, Lester C Hunt, and Baltasar Manzano. 2017. "Oil Subsidies and Renewable Energy in Saudi Arabia: A General Equilibrium Approach." *The Energy Journal* 38 (SI).
- BP. 2016. "Statistical Review of World Energy 2016."
- Calvo, Guillermo A. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12 (3). North-Holland: 383–98.
- Chang, Juin-Jen, Jhy-Hwa Chen, Jhy-Yuan Shieh, and Ching-Chong Lai. 2009. "Optimal Tax Policy, Market Imperfections, and Environmental Externalities in a Dynamic Optimizing Macro Model." *Journal of Public Economic Theory* 11 (4): 623–51.
- Christiano, Lawrence J, Martin Eichenbaum, and Charles L Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy* 113 (1). The University of Chicago Press: 1–45.
- Christiano, Lawrence J, Martin S Eichenbaum, and Mathias Trabandt. 2018. "On DSGE Models." National Bureau of Economic Research Working Paper Series No. 24811.
- Clarida, Richard, Jordi Gali, and Mark Gertler. 1999. "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature* 37 (4): 1661–1707.
- De Miguel, Carlos, and Baltasar Manzano. 2006. "Optimal Oil Taxation in a Small Open Economy." *Review of Economic Dynamics* 9 (3). Academic Press: 438–54.
- De Miguel, Carlos, Baltasar Manzano, José M Martín-Moreno, and Jesús Ruiz. 2009. "Disentangling The Effects of Oil Shocks: The Role of Rigidities and Monetary Policy." *The Energy Journal* 30. International Association for Energy Economics: 193–216.
- Dissou, Yazid, and Lilia Karnizova. 2016. "Emissions Cap or Emissions Tax? A Multi-Sector Business Cycle Analysis." *Journal of Environmental Economics and Management* 79 (September). Academic Press: 169–88.
- Economides, George, and Anastasios Xepapadeas. 2018. "Monetary Policy Under Climate Change." 7021. CESifo Working Paper.

References

- Edge, Rochelle M, Refets S Gürkaynak, Ricardo Reis, and Christopher A Sims. 2010. "How Useful Are Estimated DSGE Model Forecasts for Central Bankers?" *Brookings Papers on Economic Activity*. Brookings Institution Press, 209–59.
- Electricity and Cogeneration Regulatory Authority (ECRA). 2017. "Statistical Booklet." <http://www.ecra.gov.sa>.
- Energy Information Administration (EIA). 2017. "Country Analysis Brief: Saudi Arabia." <https://www.eia.gov/beta/international/analysis.php?iso=SAU>
- Fischer, Carolyn, and Michael Springborn. 2011. "Emissions Targets and the Real Business Cycle: Intensity Targets versus Caps or Taxes." *Journal of Environmental Economics and Management* 62 (3). Academic Press: 352–66.
- Gately, Dermot, Nourah Al-Yousef, and Hamad M.H. Al-Sheikh. 2012. "The Rapid Growth of Domestic Oil Consumption in Saudi Arabia and the Opportunity Cost of Oil Exports Foregone." *Energy Policy* 47 (August). Elsevier: 57–68.
- General Authority for Statistics (GaStat). 2016. "Supply-Use Tables." KAPSARC Data Portal. <https://datasource.kapsarc.org>.
- Government of Saudi Arabia. 2016. "Saudi Vision 2030." <http://vision2030.gov.sa/en>.
- Heutel, Garth. 2012. "How Should Environmental Policy Respond to Business Cycles? Optimal Policy under Persistent Productivity Shocks." *Review of Economic Dynamics* 15 (2). Academic Press: 244–64.
- Holladay, J. Scott, Mohammed Mohsin, and Shreekar Pradhan. 2018. "Emissions Leakage, Environmental Policy and Trade Frictions." *Journal of Environmental Economics and Management* 88 (March). Academic Press: 95–113.
- The Intergovernmental Panel on Climate Change (IPCC). 1996. "Revised 1996 IPCC Guidelines for National Greenhouse Gas Inventories." Paris: Intergovernmental Panel on Climate Change.
- Kim, In-Moo, and Prakash Loungani. 1992. "The Role of Energy in Real Business Cycle Models." *Journal of Monetary Economics* 29 (2). North-Holland: 173–89.
- King, Robert G., and Sergio T. Rebelo. 1999. "Resuscitating Real Business Cycles." In *Handbook of Macroeconomics*, edited by J. B. Taylor and M. Woodford, 927–1007. Elsevier Ltd.
- Koetse, Mark J., Henri L.F. de Groot, and Raymond J.G.M. Florax. 2008. "Capital-Energy Substitution and Shifts in Factor Demand: A Meta-Analysis." *Energy Economics* 30 (5). North-Holland: 2236–51.
- Kydland, Finn E, and Edward C Prescott. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica* 50 (6). Wiley, Econometric Society: 1345–70.
- Lin, Boqiang, and Aijun Li. 2012. "Impacts of Removing Fossil Fuel Subsidies on China: How Large and How to Mitigate?" *Energy* 44 (1). Pergamon: 741–49.
- Lindé, Jesper. 2018. "DSGE Models: Still Useful in Policy Analysis?" *Oxford Review of Economic Policy* 34 (1–2): 269–86.
- Long, John B, and Charles I Plosser. 1983. "Real Business Cycles." *Journal of Political Economy* 91 (1). University of Chicago Press: 39–69.
- Mai, Trieu, Ryan Wiser, Debra Sandor, Gregory Brinkman, Garvin Heath, Paul Denholm, Donna J. Hostick, Naim Darghouth, Adam Schollosser, and Ken Strzepek. 2012. "Exploration of High-Penetration Renewable Electricity Futures." 1. Renewable Electricity Futures Study. Golden, CO.

- Matar, Walid, Frederic Murphy, Axel Pierru, and Bertrand Rioux. 2015. "Lowering Saudi Arabia's Fuel Consumption and Energy System Costs without Increasing End Consumer Prices." *Energy Economics* 49 (May). North-Holland: 558–69.
- Pierru, Axel, and Walid Matar. 2014. "The Impact of Oil Price Volatility on Welfare in the Kingdom of Saudi Arabia: Implications for Public Investment Decision-Making." *The Energy Journal* 35 (2). International Association for Energy Economics: 97–116.
- Pierru, Axel, James L. Smith, and Tamim Zamrik. 2018. "OPEC's Impact on Oil Price Volatility: The Role of Spare Capacity." *The Energy Journal* 39 (2): 103–22.
- Plante, Michael. 2014. "The Long-Run Macroeconomic Impacts of Fuel Subsidies." *Journal of Development Economics* 107 (March). North-Holland: 129–43.
- Rotemberg, Julio J, and Michael Woodford. 1996. "Imperfect Competition and the Effects of Energy Price Increases on Economic Activity." *Journal of Money, Credit and Banking* 28 (4). Ohio State University Press: 549–77.
- Saudi Arabian Monetary Authority (SAMA). 2017. "Annual Statistics 2016." Riyadh.
- . 2018. "54th Annual Report 1439H -2018." Riyadh.
- Saudi Electricity Company (SEC). 2018. "Tariff Rates." <https://www.se.com.sa/en-us/customers/Pages/TariffRates.aspx>
- Sbordone, Argia M., Andrea Tambalotti, Krishna Rao, and Kieran Walsh. 2010. "Policy Analysis Using DSGE Models: An Introduction." *Economic Policy Review* 16 (2). Federal Reserve Bank of New York.
- Schwanitz, Valeria Jana, Franziska Piontek, Christoph Bertram, and Gunnar Luderer. 2014. "Long-Term Climate Policy Implications of Phasing out Fossil Fuel Subsidies." *Energy Policy* 67 (April). Elsevier: 882–94.
- Serletis, Apostolos, Govinda R. Timilsina, and Olexandr Vasetsky. 2010. "Interfuel Substitution in the United States." *Energy Economics* 32 (3). North-Holland: 737–45.
- Smets, Frank, and Rafael Wouters. 2003. "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area." *Journal of the European Economic Association* 1 (5): 1123–75.
- . 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review* 97 (3): 586–606.
- Wogan, David, Shreekar Pradhan, and Shahad Albardi. 2017. "GCC Energy System Overview - 2017." KAPSARC Methodology Paper KS-2017 (MP04). King Abdullah Petroleum Studies and Research Center.
- Woodford, Michael, and Carl E Walsh. 2005. "Interest and Prices: Foundations of a Theory Of Monetary Policy." *Macroeconomic Dynamics* 9 (3): 462–68.
- World Bank. 2018. "World Development Indicators." <http://data.worldbank.org/data-catalog/world-development-indicators>
- World Resources Institute (WRI). 2015. "CAIT Climate Data Explorer." Washington, DC.
- Yun, Tack. 1996. "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles." *Journal of Monetary Economics* 37 (2). North-Holland: 345–70.

Appendix I: List of notations, variables and parameters

As a general rule, subscripts refer to produced goods and services, primary inputs and natural resources (in uppercase letters); superscripts refer to categories or types of economic agents (in lowercase letters).

Table I.1. Sectors and agents

Sectors		Agents	
T	Tradable	s	Saudi
NT	Non-tradable	ns	Non-Saudi (expatriates)
ES	Energy services	g	Government
EL	Electricity	d	Domestic
O	Oil	*	Foreign
G	Natural gas		
R	Renewable energy (PV solar)		
ss	Steady state		

Source: KAPSARC.

Table I.2. Variables

Endogenous variables		Exogenous variables
1	τ_C	τ_T
2	P	τ_{NT}
3	P_T	τ_{ES}
4	P_{NT}	τ_W^s
5	I^s	τ_W^{ns}
6	I_T^s	τ_K
7	I_{NT}^s	τ_{REM}
8	I_T^{ns}	GC
9	I_{NT}^{ns}	I_g^s
10	W	I^{ns}
11	W_T	x

Endogenous variables		Exogenous variables
12	W_{NT}	i
13	MUC^s	W_g^s
14	$MUES^s$	P_O^d
15	MUL^s	P_G^d
16	C^s	P_O^*
17	ES^s	P_G^*
18	C^{ns}	I_R^d
19	ES^{ns}	ϵ_T
20	t^*	ϵ_{NT}
21	i_T	ϵ_O
22	i_{NT}	ϵ_G
23	P_{ES}	u
24	q_T	I_O^g
25	q_{NT}	P_T^*
26	I_T	
27	I_{NT}	
28	K_T	
29	K_{NT}	
30	B^*	
31	TR	
32	REM	
33	y_T	
34	y_{NT}	
35	ES_T	
36	ES_{NT}	
37	z_T	

Appendix I: List of notations, variables and parameters

Endogenous variables		Exogenous variables
38	z_{NT}	
39	EL	
40	O_{EL}	
41	G_{EL}	
42	O_{ES}	
43	G_{ES}	
44	REN	
45	P_{EL}	
46	P_R	
47	K_R^g	
48	K_O^g	
49	O^-	
50	G^-	
51	B	
52	ES	
53	GDP	
54	$P_{O,ss}^*$	
55	OX	
56	CO_2	
57	TB	

Source: KAPSARC.

Table I.3. Parameters

The model parameters are given below:

List of parameters

Aggregation of tradable and non-tradable goods consumption	
θ_c	Share of tradable goods
σ_c	Elasticity of substitution between consumption of tradable and non-tradable goods
Aggregation of tradable and non-tradable goods labor supply	
θ_L	Share of tradable goods
σ_L	Elasticity of substitution between tradable and non-tradable goods labor supply
Labor force	
$xpat$	Share of non-Saudi labor force
Saudi household	
h_T	Adjustment cost in the tradable goods sector
h_{NT}	Adjustment cost in the non-tradable goods sector
δ_T	Depreciation rate for capital of the tradable final goods
δ_{NT}	Depreciation rate for capital of the non-tradable final goods
β	Discount rate
σ	Elasticity of substitution between consumption and energy services
κ	Weight of household's energy services
α_{GC}	Weight of public services to household
η	Weight of labor disutility
ν	Inverse of Frisch labor supply elasticity
Non-Saudi household	
φ_c	Fraction of income spent on consumption goods
φ_{ES}	Fraction of income spent on energy services
Tradable goods firm	
A_T	Technology index in tradable goods production
θ_T	Share of capital in the capital-energy services aggregate
α_T	Output elasticity of labor
ν_T	Elasticity of substitution between capital and energy services

Appendix I: List of notations, variables and parameters

List of parameters

ϱ_T	Persistence of tradable goods productivity shock process
σ_T^2	Variance of innovation to tradable goods productivity shock
Non-tradable goods firm	
A_{NT}	Technology index in non-tradable goods production
θ_{NT}	Share of capital in the capital-energy services aggregate
α_{NT}	Output elasticity of labor
ν_{NT}	Elasticity of substitution parameter between capital and energy services
ϱ_{NT}	Persistence of non-tradable goods productivity shock process
σ_{NT}^2	Variance of innovation to non-tradable goods productivity shock
Energy services firm	
λ	Elasticity of substitution parameter between electricity, oil and gas
φ	Share of electricity
ψ	Share of oil
Electricity generation firm	
γ	Share of oil in fossil fuels aggregate
Natural resources: renewables	
A_R	Productivity of public capital stock in renewables production
χ	Integration cost parameter
δ_R	Depreciation rate of renewable sector capital stock
Natural resources: oil	
α_O	Productivity of public capital stock in oil production
δ_O	Depreciation rate of oil sector capital stock
ϱ_O	Persistency of oil price process
$P_{O,ss}^*$	Long-run oil price in foreign currency
σ_O^2	Variance of innovation to oil price process
ρ	Market share of Saudi oil in global oil output

List of parameters

ε_{LR}^D	Long-run price elasticity of global oil demand
ε_{LR}^S	Long-run price elasticity of supply of non-Saudi oil
ε_{SR}	Short-run price elasticity of Saudi oil exports
Natural resources: gas	
α_G	Proportionality factor between oil and gas production
ϱ_G	Persistency of gas price process
$P_{G,ss}^*$	Long-run natural gas price in foreign currency
σ_G^2	Variance of innovation to gas price process
Risk premium	
i^{**}	Risk-free international interest rate
ζ	Risk premium parameter
Carbon dioxide emissions	
ef_O	Emissions factor for oil use
ef_G	Emissions factor for gas use

Source: KAPSARC.

Appendix II: Detailed derivation of equations

A. Aggregation of tradable and non-tradable goods

A.1 Demand aggregation without taxes

The aggregator agent maximizes its consumption of final consumption goods by choosing an optimal combination of tradable goods (T) and non-tradable goods (NT) subject to its budget constraint. Assuming a CES aggregator function, the agent's problem is to maximize:

$$(A.1) \quad C_t = \left[\theta_c \frac{1}{\sigma_c} C_{T,t}^{\frac{\sigma_c-1}{\sigma_c}} + (1 - \theta_c) \frac{1}{\sigma_c} C_{NT,t}^{\frac{\sigma_c-1}{\sigma_c}} \right]^{\frac{\sigma_c}{\sigma_c-1}}$$

subject to:

$$(A.2) \quad P_{T,t} C_{T,t} + P_{NT,t} C_{NT,t} = Z_t$$

where Z is the agents' income.

Setting up the Lagrangean and letting λ be the multiplier associated with the budget constraint, the first order condition for C_T is:

$$(A.3) \quad \left(\frac{\sigma_c}{\sigma_c-1} \right) \left[\theta_c \frac{1}{\sigma_c} C_{T,t}^{\frac{\sigma_c-1}{\sigma_c}} + (1 - \theta_c) \frac{1}{\sigma_c} C_{NT,t}^{\frac{\sigma_c-1}{\sigma_c}} \right]^{\frac{\sigma_c}{\sigma_c-1}-1} \left(\frac{\sigma_c-1}{\sigma_c} \right) \theta_c \frac{1}{\sigma_c} C_{T,t}^{\frac{\sigma_c-1}{\sigma_c}-1} = \lambda_t P_{T,t}$$

Simplifying and using (A.1) into (A.3) yields:

$$(A.4) \quad \theta_c \frac{1}{\sigma_c} C_{T,t}^{\frac{1}{\sigma_c}} C_{NT,t}^{\frac{1}{\sigma_c}} = \lambda_t P_{T,t}$$

Similarly for C_{NT} :

$$(A.5) \quad (1 - \theta_c) \frac{1}{\sigma_c} C_{NT,t}^{\frac{1}{\sigma_c}} C_{T,t}^{\frac{1}{\sigma_c}} = \lambda_t P_{NT,t}$$

Take now the ratio between (A.4) and (A.5):

$$(A.6) \quad \left(\frac{\theta_c}{1 - \theta_c} \right)^{\frac{1}{\sigma_c}} \left(\frac{C_{T,t}}{C_{NT,t}} \right)^{-\frac{1}{\sigma_c}} = \frac{P_{T,t}}{P_{NT,t}}$$

and rearrange to get:

$$(A.7) \quad C_{T,t} = \left(\frac{P_{T,t}}{P_{NT,t}} \right)^{-\sigma_c} \left(\frac{\theta_c}{1 - \theta_c} \right) C_{NT,t}$$

Use (A.7) in the budget constraint (A.2):

$$\left(\frac{P_{T,t}}{P_{NT,t}} \right)^{-\sigma_c} \left(\frac{\theta_c}{1 - \theta_c} \right) P_{T,t} C_{NT,t} + P_{NT,t} C_{NT,t} = Z_t$$

Solve for C_{NT} :

$$(A.8) \quad C_{NT,t} = \frac{Z_t}{P_{T,t}^{-\sigma_C} P_{NT,t}^{\sigma_C} \left(\frac{\theta_C}{1-\theta_C} \right) P_{T,t} + P_{NT,t}} = \frac{(1-\theta_C) P_{NT,t}^{-\sigma_C} Z_t}{\theta_C P_{T,t}^{1-\sigma_C} + (1-\theta_C) P_{NT,t}^{1-\sigma_C}}$$

Similarly:

$$(A.9) \quad C_T = \frac{\theta_C P_{T,t}^{-\sigma_C} Z_t}{\theta_C P_{T,t}^{1-\sigma_C} + (1-\theta_C) P_{NT,t}^{1-\sigma_C}}$$

Plug (A.8)-(A.9) into the CES consumption aggregator (A.1):

$$(A.10) \quad C_t = \left[\theta_C^{\frac{1}{\sigma_C}} \left(\frac{\theta_C P_{T,t}^{-\sigma_C} Z_t}{\theta_C P_{T,t}^{1-\sigma_C} + (1-\theta_C) P_{NT,t}^{1-\sigma_C}} \right)^{\frac{\sigma_C-1}{\sigma_C}} + (1-\theta_C)^{\frac{1}{\sigma_C}} \left(\frac{(1-\theta_C) P_{NT,t}^{-\sigma_C} Z_t}{\theta_C P_{T,t}^{1-\sigma_C} + (1-\theta_C) P_{NT,t}^{1-\sigma_C}} \right)^{\frac{\sigma_C-1}{\sigma_C}} \right]^{\frac{\sigma_C}{\sigma_C-1}}$$

$$= \left[\frac{Z_t}{\theta_C P_{T,t}^{1-\sigma_C} + (1-\theta_C) P_{NT,t}^{1-\sigma_C}} \right] \left[\theta_C P_{T,t}^{1-\sigma_C} + (1-\theta_C) P_{NT,t}^{1-\sigma_C} \right]^{\frac{\sigma_C}{\sigma_C-1}}$$

$$= Z_t \left[\theta_C P_{T,t}^{1-\sigma_C} + (1-\theta_C) P_{NT,t}^{1-\sigma_C} \right]^{\frac{1}{\sigma_C-1}}$$

The total expenditure needed to buy a single unit of consumption is the price index P ,

i.e., $Z/C = PC/C = P$.

From (A.10):

$$(A.11) \quad P_t = \left[\theta_C P_{T,t}^{1-\sigma_C} + (1-\theta_C) P_{NT,t}^{1-\sigma_C} \right]^{\frac{1}{1-\sigma_C}}$$

which is a CES price aggregator function.

In the case of a Cobb-Douglas aggregator function ($\sigma_C=1$) the price aggregator is

$$P_t = P_{T,t}^{\theta_C} P_{NT,t}^{1-\theta_C}$$

Take (A.7) and plug it into the budget constraint:

$$Z_t = P_{T,t} C_{T,t} + P_{NT,t} \left(\frac{P_{NT,t}}{P_{T,t}} \right)^{-\sigma_C} \left(\frac{1-\theta_C}{\theta_C} \right) C_{T,t}$$

$$= P_{T,t}^{\sigma_C} \frac{C_T}{\theta_C} \left[\theta_C P_{T,t}^{1-\sigma_C} + (1-\theta_C) P_{NT,t}^{1-\sigma_C} \right]$$

$$= P_{T,t}^{\sigma_C} \frac{C_T}{\theta_C} P_t^{1-\sigma_C}$$

So that:

$$C_{T,t} = \theta_C \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma_C} \frac{Z_t}{P_t}$$

Appendix II: Detailed derivation of equations

Because $Z=PC$:

$$(A.12) \quad C_{T,t} = \theta_C \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma_C} C_t$$

and:

$$(A.13) \quad C_{NT,t} = (1 - \theta_C) \left(\frac{P_{NT,t}}{P_t} \right)^{-\sigma_C} C_t$$

Finally note that:

$$\begin{aligned} & P_{T,t} C_{T,t} + P_{NT,t} C_{NT,t} \\ &= P_{T,t} \theta_C \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma_C} C_t + P_{NT,t} (1 - \theta_C) \left(\frac{P_{NT,t}}{P_t} \right)^{-\sigma_C} C_t \\ &= P_t^{\sigma_C} C_t [\theta_C P_{T,t}^{1-\sigma_C} + (1 - \theta_C) P_{NT,t}^{1-\sigma_C}] \\ &= P_t^{\sigma_C} P^{1-\sigma_C} C \end{aligned}$$

We conclude that:

$$(A.14) \quad P_t C_t = P_{T,t} C_{T,t} + P_{NT,t} C_{NT,t}$$

A.2 Demand aggregation with taxes

If we allow for indirect taxes on tradable and non-tradable goods, the aggregation proceeds along the same lines. Thus, let the consumption aggregator be given by (A.1). The problem is now to maximize (A.1) subject to:

$$(A.15) \quad Z_t = (1 + \tau_{T,t}) P_{T,t} C_{T,t} + (1 + \tau_{NT,t}) P_{NT,t} C_{NT,t}$$

where τ_T and τ_{NT} are tax rates on the types of consumption goods. The first order condition for C_T is:

$$\begin{aligned} (A.16) \quad & \left(\frac{\sigma_C}{\sigma_C - 1} \right) \left[\theta_C^{\frac{1}{\sigma_C}} C_{T,t}^{\frac{\sigma_C - 1}{\sigma_C}} + (1 - \theta_C)^{\frac{1}{\sigma_C}} C_{NT,t}^{\frac{\sigma_C - 1}{\sigma_C}} \right]^{\frac{\sigma_C}{\sigma_C - 1} - 1} \left(\frac{\sigma_C - 1}{\sigma_C} \right) \theta_C^{\frac{1}{\sigma_C}} C_{T,t}^{\frac{\sigma_C - 1}{\sigma_C} - 1} \\ &= \left[\theta_C^{\frac{1}{\sigma_C}} C_{T,t}^{\frac{\sigma_C - 1}{\sigma_C}} + (1 - \theta_C)^{\frac{1}{\sigma_C}} C_{NT,t}^{\frac{\sigma_C - 1}{\sigma_C}} \right]^{\frac{1}{\sigma_C - 1}} \theta_C^{\frac{1}{\sigma_C}} C_{T,t}^{\frac{1}{\sigma_C} - 1} \\ &= \lambda_t (1 + \tau_{T,t}) P_{T,t} \end{aligned}$$

Simplifying and using (A.1) into (A.16) yields:

$$(A.17) \quad C_t^{\frac{1}{\sigma_C}} \theta_C^{\frac{1}{\sigma_C}} C_{T,t}^{\frac{1}{\sigma_C} - 1} = \lambda_t (1 + \tau_{T,t}) P_{T,t}$$

Similarly:

$$(A.18) \quad C_t^{\frac{1}{\sigma_C}} (1 - \theta_C)^{\frac{1}{\sigma_C}} C_{NT,t}^{\frac{1}{\sigma_C} - 1} = \lambda_t (1 + \tau_{NT,t}) P_{NT,t}$$

Take now the ratio between (A.17) and (A.18) and rearrange to get:

$$(A.19) \quad C_{T,t} = \left[\frac{(1+\tau_{T,t})P_{T,t}}{(1+\tau_{NT,t})P_{NT,t}} \right]^{-\sigma_C} \left(\frac{\theta_C}{1-\theta_C} \right) C_{NT,t}$$

Use (A.19) in the budget constraint (A.15):

$$Z_t = \left[\frac{(1+\tau_{T,t})P_{T,t}}{(1+\tau_{NT,t})P_{NT,t}} \right]^{-\sigma_C} \left(\frac{\theta_C}{1-\theta_C} \right) (1+\tau_{T,t})P_{T,t}C_{NT,t} + (1+\tau_{NT,t})P_{NT,t}C_{NT,t}$$

Solve for $C_{NT,t}$:

$$C_{NT,t} = \frac{Z_t}{\left(\frac{\theta_C}{1-\theta_C} \right) (1+\tau_{T,t})P_{T,t} \left[\frac{(1+\tau_{T,t})P_{T,t}}{(1+\tau_{NT,t})P_{NT,t}} \right]^{-\sigma_C} + (1+\tau_{NT,t})P_{NT,t}}$$

or:

$$(A.20) \quad C_{NT,t} = \frac{(1-\theta_C)Z_t[(1+\tau_{NT,t})P_{NT,t}]^{-\sigma_C}}{\theta_C[(1+\tau_{T,t})P_{T,t}]^{1-\sigma_C} + (1-\theta_C)[(1+\tau_{NT,t})P_{NT,t}]^{1-\sigma_C}}$$

Similarly:

$$(A.21) \quad C_{T,t} = \frac{\theta_C Z_t [(1+\tau_{T,t})P_{T,t}]^{-\sigma_C}}{\theta_C [(1+\tau_{T,t})P_{T,t}]^{1-\sigma_C} + (1-\theta_C) [(1+\tau_{NT,t})P_{NT,t}]^{1-\sigma_C}}$$

Plug (A.20)-(A.21) into the CES consumption aggregator (A.1):

$$(A.22) \quad C_t = \left[\theta_C^{\frac{1}{\sigma_C}} \left(\frac{\theta_C Z_t [(1+\tau_{T,t})P_{T,t}]^{-\sigma_C}}{\theta_C [(1+\tau_{T,t})P_{T,t}]^{1-\sigma_C} + (1-\theta_C) [(1+\tau_{NT,t})P_{NT,t}]^{1-\sigma_C}} \right)^{\frac{\sigma_C-1}{\sigma_C}} + (1 - \theta_C)^{\frac{1}{\sigma_C}} \left(\frac{(1-\theta_C) Z_t [(1+\tau_{NT,t})P_{NT,t}]^{-\sigma_C}}{\theta_C [(1+\tau_{T,t})P_{T,t}]^{1-\sigma_C} + (1-\theta_C) [(1+\tau_{NT,t})P_{NT,t}]^{1-\sigma_C}} \right)^{\frac{\sigma_C-1}{\sigma_C}} \right]^{\frac{\sigma_C}{\sigma_C-1}} \\ = \left[\frac{Z_t}{\theta_C [(1+\tau_{T,t})P_{T,t}]^{1-\sigma_C} + (1-\theta_C) [(1+\tau_{NT,t})P_{NT,t}]^{1-\sigma_C}} \right] \left[\theta_C [(1+\tau_{T,t})P_{T,t}]^{1-\sigma_C} + (1-\theta_C) [(1+\tau_{NT,t})P_{NT,t}]^{1-\sigma_C} \right]^{\frac{\sigma_C}{\sigma_C-1}} \\ = Z_t \left\{ \theta_C [(1+\tau_{T,t})P_{T,t}]^{1-\sigma_C} + (1-\theta_C) [(1+\tau_{NT,t})P_{NT,t}]^{1-\sigma_C} \right\}^{\frac{1}{1-\sigma_C}}$$

The total expenditure needed to buy a single unit of consumption is the after tax price index

$(1+\tau_{C,t})P_t$, i.e. $Z_t/C_t = (1+\tau_{C,t})P_t C_t/C_t = (1+\tau_{C,t})P_t$. From (A.22):

$$(A.23) \quad (1+\tau_{C,t})P_t = \left\{ \theta_C [(1+\tau_{T,t})P_{T,t}]^{1-\sigma_C} + (1-\theta_C) [(1+\tau_{NT,t})P_{NT,t}]^{1-\sigma_C} \right\}^{\frac{1}{1-\sigma_C}}$$

which is a CES price aggregator function. In the case of a Cobb-Douglas aggregator function ($\sigma_C=1$) the price aggregator is

$$(A.24) \quad (1+\tau_{C,t})P_t = \left((1+\tau_{T,t})P_{T,t} \right)^{\theta_C} \left((1+\tau_{NT,t})P_{NT,t} \right)^{1-\theta_C}$$

Appendix II: Detailed derivation of equations

Take (A.19) and plug it into the budget constraint:

$$\begin{aligned}
 Z_t &= (1 + \tau_{T,t})P_{T,t}C_{T,t} + (1 + \tau_{NT,t})P_{NT,t} \left[\frac{(1 + \tau_{NT,t})P_{NT,t}}{(1 + \tau_{C,t})P_t} \right]^{-\sigma_C} \left(\frac{1 - \theta_C}{\theta_C} \right) C_{T,t} \\
 &= [(1 + \tau_{T,t})P_{T,t}]^{\sigma_C} \left(\frac{C_{T,t}}{\theta_C} \right) \left\{ \theta_C [(1 + \tau_{T,t})P_{T,t}]^{1 - \sigma_C} + (1 - \theta_C) [(1 + \tau_{NT,t})P_{NT,t}]^{1 - \sigma_C} \right\} \\
 &= [(1 + \tau_{T,t})P_{T,t}]^{\sigma_C} \left(\frac{C_{T,t}}{\theta_C} \right) [(1 + \tau_{C,t})P_t]^{1 - \sigma_C}
 \end{aligned}$$

So that:

$$C_{T,t} = \theta_C \left[\frac{(1 + \tau_{T,t})P_{T,t}}{(1 + \tau_{C,t})P_t} \right]^{-\sigma_C} \frac{Z_t}{(1 + \tau_{C,t})P_t}$$

Because $Z_t = (1 + \tau_{C,t})P_t C_t$:

$$(A.25) \quad C_{T,t} = \theta_C \left[\frac{(1 + \tau_{T,t})P_{T,t}}{(1 + \tau_{C,t})P_t} \right]^{-\sigma_C} C_t$$

and:

$$(A.26) \quad C_{NT,t} = (1 - \theta_C) \left[\frac{(1 + \tau_{NT,t})P_{NT,t}}{(1 + \tau_{C,t})P_t} \right]^{-\sigma_C} C_t$$

Finally note that:

$$\begin{aligned}
 C_{T,t} + (1 + \tau_{NT,t})P_{NT,t}C_{NT,t} &= \\
 &= \theta_C (1 + \tau_{T,t})P_{T,t} \left[\frac{(1 + \tau_{T,t})P_{T,t}}{(1 + \tau_{C,t})P_t} \right]^{-\sigma_C} C_t \\
 &\quad + (1 - \theta_C) (1 + \tau_{NT,t})P_{NT,t} \left[\frac{(1 + \tau_{NT,t})P_{NT,t}}{(1 + \tau_{C,t})P_t} \right]^{-\sigma_C} C_t \\
 &= \left\{ \theta_C [(1 + \tau_{T,t})P_{T,t}]^{1 - \sigma_C} + (1 - \theta_C) [(1 + \tau_{NT,t})P_{NT,t}]^{1 - \sigma_C} \right\} [(1 + \tau_{C,t})P_t]^{\sigma_C} C_t \\
 &= [(1 + \tau_{C,t})P_t]^{\sigma_C} [(1 + \tau_{C,t})P_t]^{1 - \sigma_C} C_t
 \end{aligned}$$

We conclude that:

$$(A.27) \quad (1 + \tau_{C,t})P_t C_t = (1 + \tau_{T,t})P_{T,t}C_{T,t} + (1 + \tau_{NT,t})P_{NT,t}C_{NT,t}$$

Now take (A.23) and divide it by the aggregate price index without taxes in (A.11):

$$(A.28a) \quad (1 + \tau_{C,t}) = \left\{ \frac{\theta_C [(1 + \tau_{T,t})P_{T,t}]^{1 - \sigma_C} + (1 - \theta_C) [(1 + \tau_{NT,t})P_{NT,t}]^{1 - \sigma_C}}{\theta_C P_{T,t}^{1 - \sigma_C} + (1 - \theta_C) P_{NT,t}^{1 - \sigma_C}} \right\}^{\frac{1}{1 - \sigma_C}}$$

In the case of Cobb-Douglas, from (A.24), we have:

$$(A.28b) \quad (1 + \tau_{C,t}) = (1 + \tau_{T,t})^{\theta_C} (1 + \tau_{NT,t})^{1 - \theta_C}$$

This implies that we can express the aggregate tax rate on consumption goods as a function of tax rates and prices of tradable and non-tradable goods, as shown in (A.28a) in the case of a CES aggregator. In the case of a Cobb-Douglas aggregator, the aggregate tax rate does not depend on prices as shown in (A.28b).

A.3 Labor supply aggregation

Following the same aggregation procedure for consumption, we have a CES aggregator for labor supply of individual workers in the tradable and non-tradable sectors that yields the following without-taxes expressions:

$$(A.29) \quad l_t = \left[\theta_L^{\frac{1}{\sigma_L}} l_{T,t}^{\frac{\sigma_L-1}{\sigma_L}} + (1-\theta_L)^{\frac{1}{\sigma_L}} l_{NT,t}^{\frac{\sigma_L-1}{\sigma_L}} \right]^{\frac{\sigma_L}{\sigma_L-1}}$$

$$(A.30) \quad l_{T,t} = \theta_L \left(\frac{W_{T,t}}{W_t} \right)^{-\sigma_L} l_t$$

$$(A.31) \quad l_{NT,t} = (1-\theta_L) \left(\frac{W_{NT,t}}{W_t} \right)^{-\sigma_L} l_t$$

$$(A.32) \quad W_t l_t = W_{T,t} l_{T,t} + W_{NT,t} l_{NT,t}$$

We now introduce taxation on labor income. It is assumed that the tax rates in the tradable and non-tradable sectors are the same. The above expressions with taxes become:

$$(1-\tau_{W,t})W_t = (1-\tau_{W,t}) \left[\theta_L (W_{T,t})^{1-\sigma_L} + (1-\theta_L) (W_{NT,t})^{1-\sigma_L} \right]^{\frac{1}{1-\sigma_L}}$$

which implies:

$$(A.33) \quad W_t = \left[\theta_L (W_{T,t})^{1-\sigma_L} + (1-\theta_L) (W_{NT,t})^{1-\sigma_L} \right]^{\frac{1}{1-\sigma_L}}$$

$$(A.34) \quad l_{T,t} = \theta_L \left[\frac{W_{T,t}}{W_t} \right]^{-\sigma_L} l_t$$

$$(A.35) \quad l_{NT,t} = (1-\theta_L) \left[\frac{W_{NT,t}}{W_t} \right]^{-\sigma_L} l_t$$

$$(A.36) \quad (1-\tau_{W,t})W_t l_t = (1-\tau_{W,t})W_{T,t} l_{T,t} + (1-\tau_{W,t})W_{NT,t} l_{NT,t}$$

B. Model

B.1 Labor force

The total labor force N is the sum of the Saudi and non-Saudi labor forces. That is:

$$(B.1) \quad N_t = N_t^s + N_t^{ns} = (1-xpat) * N_t + xpat * N_t$$

where $xpat$ is the share of non-Saudis in the total labor force. For simplicity, we take this share to be constant, although it can be made time-varying.

Appendix II: Detailed derivation of equations

B.2 Saudi household

The Saudi household's problem is to maximize the welfare, given by the expected present discounted value of an infinite stream of instantaneous utilities:

$$(B.2) \quad E_0 \sum_{t=0}^{\infty} \beta^t U^s(C_t^s, ES_t^s, l_t^s + l_{g,t}^s, GC_t)$$

subject to the budget constraint

$$(B.3) \quad \begin{aligned} & (1 + \tau_{C,t})P_t C_t^s + P_t \left[I_{T,t} + \frac{h_T}{2} \left(\frac{I_{T,t}}{K_{T,t-1}} - \delta_T \right)^2 K_{T,t-1} \right] + P_t \left[I_{NT,t} + \frac{h_{NT}}{2} \left(\frac{I_{NT,t}}{K_{NT,t-1}} - \delta_{NT} \right)^2 K_{NT,t-1} \right] \\ & + (1 + \tau_{ES,t})P_{ES,t} ES_t^s + x_t B_t^* - (x_{t-1} + x_t i_t^*) B_{t-1}^* + B_t - (1 + i_t) B_{t-1} \\ & = (1 - \tau_{W,t}^s) [W_t^s l_t^s + W_{g,t}^s l_{g,t}^s] + (1 - \tau_{K,t}) [i_{T,t} K_{T,t-1} + i_{NT,t} K_{NT,t-1}] + TR_t \end{aligned}$$

The two fixed capital stocks are as follows:

$$(B.4) \quad I_{T,t} = K_{T,t} - (1 - \delta_T) K_{T,t-1}$$

$$(B.5) \quad I_{NT,t} = K_{NT,t} - (1 - \delta_{NT}) K_{NT,t-1}$$

To maximize (B.2) subject to constraints (B.3), (B.4) and (B.5) we set up the following Lagrangean:

$$\begin{aligned} \mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ & U^s(C_t^s, ES_t^s, l_t^s + l_{g,t}^s, GC_t) + \lambda_{BC,t} \Big\{ (1 - \tau_{W,t}^s) [W_t^s l_t^s + W_{g,t}^s l_{g,t}^s] \\ & + (1 - \tau_{K,t}) [i_{T,t} K_{T,t-1} + i_{NT,t} K_{NT,t-1}] + TR_t - (1 + \tau_{C,t}) P_t C_t^s - \\ & P_t \left[I_{T,t} + \frac{h_T}{2} \left(\frac{I_{T,t}}{K_{T,t-1}} - \delta_T \right)^2 K_{T,t-1} \right] - P_t \left[I_{NT,t} + \frac{h_{NT}}{2} \left(\frac{I_{NT,t}}{K_{NT,t-1}} - \delta_{NT} \right)^2 K_{NT,t-1} \right] - \\ & (1 + \tau_{ES,t}) P_{ES,t} ES_t^s - x_t B_t^* + (x_{t-1} + x_t i_t^*) B_{t-1}^* - B_t + (1 + i_t) B_{t-1} \Big\} + \\ & \lambda_{T,t} [I_{T,t} - K_{T,t} + (1 - \delta_T) K_{T,t-1}] + \lambda_{NT,t} [I_{NT,t} - K_{NT,t} + (1 - \delta_{NT}) K_{NT,t-1}] \Big\} \end{aligned}$$

where $\lambda_{BC,t}, \lambda_{T,t}, \lambda_{NT,t}$ are the Lagrange multipliers associated with constraints (B.3), (B.4) and (B.5). The decision variables in the problem are: $C_t^s, B_t^*, B_t, ES_t^s, l_t^s$. Note that $GC_t, l_{g,t}^s$ and TR_t are exogenous policy variables.

The first order conditions are:

$$(B.6) \quad \frac{\partial U^s}{\partial C_t^s} - \lambda_{BC,t} (1 + \tau_{C,t}) P_t = 0$$

$$(B.7) \quad \frac{\partial U^s}{\partial ES_t^s} - \lambda_{BC,t} (1 + \tau_{ES,t}) P_{ES,t} = 0$$

$$(B.8) \quad \frac{\partial U^s}{\partial l_t^s} + \lambda_{BC,t} (1 - \tau_{W,t}^s) W_t^s = 0$$

$$(B.9) \quad -\lambda_{BC,t} P_t \left[1 + h_T \left(\frac{I_{T,t}}{K_{T,t-1}} - \delta_T \right) \right] + \lambda_{T,t} = 0$$

$$(B.10) \quad -\lambda_{T,t} + \beta E_t \lambda_{T,t+1} (1 - \delta_T) - \beta E_t \lambda_{BC,t+1} P_{t+1} \frac{h_T}{2} \left(\frac{I_{T,t+1}}{K_{T,t}} - \delta_T \right) \left(-\frac{I_{T,t+1}}{K_{T,t}} - \delta_T \right) + \beta E_t \lambda_{BC,t+1} (1 - \tau_{K,t+1}) i_{T,t+1} = 0$$

$$(B.11) \quad -\lambda_{BC,t}P_t \left[1 + h_{NT} \left(\frac{I_{NT,t}}{K_{NT,t-1}} - \delta_{NT} \right) \right] + \lambda_{NT,t} = 0$$

$$(B.12) \quad -\lambda_{NT,t} + \beta E_t \lambda_{NT,t+1} (1 - \delta_{NT}) - \beta E_t \lambda_{BC,t+1} P_{t+1} \frac{h_{NT}}{2} \left(\frac{I_{NT,t+1}}{K_{NT,t}} - \delta_{NT} \right) \left(-\frac{I_{NT,t+1}}{K_{NT,t}} - \delta_{NT} \right) + \beta E_t \lambda_{BC,t+1} (1 - \tau_{K,t+1}) i_{NT,t+1} = 0$$

$$(B.13) \quad -\lambda_{BC,t}x_t + \beta E_t \lambda_{BC,t+1} (x_t + x_{t+1} i_{t+1}^*) = 0$$

$$(B.14) \quad -\lambda_{BC,t} + \beta E_t \lambda_{BC,t+1} (1 + i_{t+1}) = 0$$

Consider marginal rates of substitution by dividing both (B.7) and (B.8) by (B.6) to obtain:

$$(B.15) \quad \frac{\frac{\partial U^S}{\partial ES_t^S}}{\frac{\partial U^S}{\partial C_t^S}} = \frac{(1 + \tau_{ES,t}) P_{ES,t}}{(1 + \tau_{C,t}) P_t}$$

$$(B.16) \quad \frac{-\frac{\partial U^S}{\partial I_t^S}}{\frac{\partial U^S}{\partial C_t^S}} = \frac{(1 - \tau_{W,t}^S) W_t^S}{(1 + \tau_{C,t}) P_t}$$

so that marginal rates of substitution are equal to relative prices.

To simplify the optimality conditions for investment and capital stock, let $q_{T,t} = \lambda_{T,t} / \lambda_{BC,t}$ and $q_{NT,t} = \lambda_{NT,t} / \lambda_{BC,t}$. Divide (B.9) and (B.11) by $\lambda_{BC,t}$ to get:

$$(B.17) \quad q_{T,t} = P_t \left[1 + h_T \left(\frac{I_{T,t}}{K_{T,t-1}} - \delta_T \right) \right]$$

$$(B.18) \quad q_{NT,t} = P_t \left[1 + h_{NT} \left(\frac{I_{NT,t}}{K_{NT,t-1}} - \delta_{NT} \right) \right]$$

Consider now the optimality conditions for the capital stocks (B.10) and (B.12): divide each expression throughout by $\lambda_{BC,t}$ and recall that $q_{T,t} = \lambda_{T,t} / \lambda_{BC,t}$ and $q_{NT,t} = \lambda_{NT,t} / \lambda_{BC,t}$. Thus:

$$(B.19) \quad q_{T,t} = \beta E_t \left[\frac{\lambda_{T,t+1}}{\lambda_{BC,t}} (1 - \delta_T) + \left(\frac{\lambda_{BC,t+1}}{\lambda_{BC,t}} \right) (1 - \tau_{K,t+1}) i_{T,t+1} - \left(\frac{\lambda_{BC,t+1}}{\lambda_{BC,t}} \right) P_{t+1} \frac{h_T}{2} \left(\frac{I_{T,t+1}}{K_{T,t}} - \delta_T \right) \left(-\frac{I_{T,t+1}}{K_{T,t}} - \delta_T \right) \right]$$

$$(B.20) \quad q_{NT,t} = \beta E_t \left[\frac{\lambda_{NT,t+1}}{\lambda_{BC,t}} (1 - \delta_{NT}) + \left(\frac{\lambda_{BC,t+1}}{\lambda_{BC,t}} \right) (1 - \tau_{K,t+1}) i_{NT,t+1} - \left(\frac{\lambda_{BC,t+1}}{\lambda_{BC,t}} \right) P_{t+1} \frac{h_{NT}}{2} \left(-\frac{I_{NT,t+1}}{K_{NT,t}} - \delta_{NT} \right) \left(-\frac{I_{NT,t+1}}{K_{NT,t}} - \delta_{NT} \right) \right]$$

Consider now the optimality condition (B.13), divide by $\lambda_{BC,t}$ and solve as follows:

$$(B.21) \quad \beta E_t \left(\frac{\lambda_{BC,t+1}}{\lambda_{BC,t}} \right) = E_t \frac{x_t}{x_t + x_{t+1} i_{t+1}^*} = E_t \frac{1}{1 + \frac{x_{t+1} i_{t+1}^*}{x_t}}$$

(B.21) into (B.19)-(B.20):

$$(B.22) \quad q_{T,t} = E_t \frac{1}{1 + \frac{x_{t+1} i_{t+1}^*}{x_t}} \left[q_{T,t+1} (1 - \delta_T) + (1 - \tau_{K,t+1}) i_{T,t+1} - P_{t+1} \frac{h_T}{2} \left(\frac{I_{T,t+1}}{K_{T,t}} - \delta_T \right) \left(-\frac{I_{T,t+1}}{K_{T,t}} - \delta_T \right) \right]$$

$$(B.23) \quad q_{NT,t} = E_t \frac{1}{1 + \frac{x_{t+1} i_{t+1}^*}{x_t}} \left[q_{NT,t+1} (1 - \delta_{NT}) + (1 - \tau_{K,t+1}) i_{NT,t+1} - P_{t+1} \frac{h_{NT}}{2} \left(\frac{I_{NT,t+1}}{K_{NT,t}} - \delta_{NT} \right) \left(-\frac{I_{NT,t+1}}{K_{NT,t}} - \delta_{NT} \right) \right]$$

Appendix II: Detailed derivation of equations

Finally, consider the optimality conditions on foreign and government debts (B.13)-(B.14). Use condition (B.6) to eliminate the Lagrange multiplier $\lambda_{BC,t}$ to obtain:

$$(B.24) \quad \frac{\frac{\partial U^s}{\partial C_t^s}}{(1+\tau_{C,t})P_t} x_t = \beta E_t \frac{\frac{\partial U^s}{\partial C_{t+1}^s}}{(1+\tau_{C,t+1})P_{t+1}} (x_t + x_{t+1} i_{t+1}^*)$$

$$(B.25) \quad \frac{\frac{\partial U^s}{\partial C_t^s}}{(1+\tau_{C,t})P_t} = \beta E_t \frac{\frac{\partial U^s}{\partial C_{t+1}^s}}{(1+\tau_{C,t+1})P_{t+1}} (1 + i_{t+1})$$

The next step is to parametrize the utility function in (B.2). Let:

$$(B.26) \quad U^s(C_t^s, ES_t^s, l_t^s, GC_t) = \ln\{[(C_t^s)^\sigma + \kappa(ES_t^s)^\sigma]^{1/\sigma} + \alpha_{CG} GC_t\} - \left(\frac{\eta}{1+\nu}\right) (l_t^s + l_{g,t}^s)^{1+\nu}$$

Marginal utilities are:

$$(B.27) \quad \frac{\partial U^s}{\partial C_t^s} = \frac{(C_t^s)^{\sigma-1} [(C_t^s)^\sigma + \kappa(ES_t^s)^\sigma]^{\frac{1-\sigma}{\sigma}}}{[(C_t^s)^\sigma + \kappa(ES_t^s)^\sigma]^{1/\sigma} + \alpha_{CG} GC_t}$$

$$(B.28) \quad \frac{\partial U^s}{\partial ES_t^s} = \frac{\kappa(ES_t^s)^{\sigma-1} [(C_t^s)^\sigma + \kappa(ES_t^s)^\sigma]^{\frac{1-\sigma}{\sigma}}}{[(C_t^s)^\sigma + \kappa(ES_t^s)^\sigma]^{1/\sigma} + \alpha_{CG} GC_t}$$

$$(B.29) \quad \frac{\partial U^s}{\partial l_{t,t}^s} = -\eta (l_t^s + l_{g,t}^s)^\nu$$

On this basis, the optimality conditions (B.15)-(B.16) and (B.24)-(B.25) become:

$$(B.30) \quad \kappa \left(\frac{ES_t^s}{C_t^s} \right)^{\sigma-1} = \frac{(1+\tau_{ES,t})P_{ES,t}}{(1+\tau_{C,t})P_t}$$

$$(B.31) \quad \frac{\eta (l_t^s)^\nu [(C_t^s)^\sigma + \kappa(ES_t^s)^\sigma]^{1/\sigma} + \alpha_{CG} GC_t}{(C_t^s)^{\sigma-1} [(C_t^s)^\sigma + \kappa(ES_t^s)^\sigma]^{\frac{1-\sigma}{\sigma}}} = \frac{(1-\tau_{W,t}^s)W_t^s}{(1+\tau_{C,t})P_t}$$

$$(B.32) \quad x_t \left(\frac{1}{(1+\tau_{C,t})P_t} \right) \left\{ \frac{(C_t^s)^{\sigma-1} [(C_t^s)^\sigma + \kappa(ES_t^s)^\sigma]^{\frac{1-\sigma}{\sigma}}}{[(C_t^s)^\sigma + \kappa(ES_t^s)^\sigma]^{1/\sigma} + \alpha_{CG} GC_t} \right\} =$$

$$\beta E_t (x_t + x_{t+1} i_{t+1}^*) \left(\frac{1}{(1+\tau_{C,t+1})P_{t+1}} \right) \left\{ \frac{(C_{t+1}^s)^{\sigma-1} [(C_{t+1}^s)^\sigma + \kappa(ES_{t+1}^s)^\sigma]^{\frac{1-\sigma}{\sigma}}}{[(C_{t+1}^s)^\sigma + \kappa(ES_{t+1}^s)^\sigma]^{1/\sigma} + \alpha_{CG} GC_{t+1}} \right\}$$

$$(B.33) \quad \left(\frac{1}{(1+\tau_{C,t})P_t} \right) \left\{ \frac{(C_t^s)^{\sigma-1} [(C_t^s)^\sigma + \kappa(ES_t^s)^\sigma]^{\frac{1-\sigma}{\sigma}}}{[(C_t^s)^\sigma + \kappa(ES_t^s)^\sigma]^{1/\sigma} + \alpha_{CG} GC_t} \right\} =$$

$$\beta E_t (1 + i_{t+1}) \left(\frac{1}{(1+\tau_{C,t+1})P_{t+1}} \right) \left\{ \frac{(C_{t+1}^s)^{\sigma-1} [(C_{t+1}^s)^\sigma + \kappa(ES_{t+1}^s)^\sigma]^{\frac{1-\sigma}{\sigma}}}{[(C_{t+1}^s)^\sigma + \kappa(ES_{t+1}^s)^\sigma]^{1/\sigma} + \alpha_{CG} GC_{t+1}} \right\}$$

In sum, the solution of the problem of maximizing (B.2) subject to the constraints (B.3)-(B.5) is given by the optimality conditions (B.30)-(B.33), (B.17)-(B.18) and (B.22)-(B.23), together with definition (B.15) and the constraints (B.3)-(B.5).

B.3 Non-Saudi household

The non-Saudi household's labor supply l_t^{ns} is assumed to be exogenous. We also assume that non-Saudi workers are not employed by the public sector. The only source of expatriates' incomes is labor. Thus, their net income is

$$(B.34) \quad (1 - \tau_{W,t}^{ns}) W_t^{ns} l_t^{ns}$$

It is assumed that a constant fraction of income is spent on consumption goods and on energy services, while the rest is saved and transferred abroad as remittances. Let the fractions be given by φ_C and φ_{ES} respectively. Thus:

$$(B.35) \quad (1 + \tau_{C,t})P_t C_t^{ns} = \varphi_C (1 - \tau_{w,t}^{ns}) W_t^{ns} l_t^{ns}$$

$$(B.36) \quad (1 + \tau_{ES,t})P_{ES,t} ES_t^{ns} = \varphi_{ES} (1 - \tau_{w,t}^{ns}) W_t^{ns} l_t^{ns}$$

$$(B.37) \quad (1 + \tau_{REM,t})REM_t = (1 - \varphi_C - \varphi_{ES})(1 - \tau_{w,t}^{ns}) W_t^{ns} l_t^{ns}$$

This specification implies that the non-Saudi household does not optimize and the sources of income and expenditure are exogenously determined. An important reason for this assumption is the lack of necessary data with which to calibrate the model.

B.4 Tradable goods firm

The production function of the tradable goods firm is the following:

$$(B.38) \quad Y_{T,t} = z_{T,t} A_T [L_{T,t}^s + L_{T,t}^{ns}]^{\alpha_T} [\theta_T K_{T,t-1}^{-\nu_T} + (1 - \theta_T) ES_{T,t}^{-\nu_T}]^{\frac{-(1-\alpha_T)}{\nu_T}}$$

where K and ES are the fixed capital and energy services inputs. All variables are in aggregate (total) terms. A_T is a technology progress indicator and $z_{T,t}$ is a shock to productivity evolving as an AR(1) process:

$$(B.39) \quad \ln z_{T,t} = \rho_T \ln z_{T,t-1} + \epsilon_{T,t} \quad \text{with} \quad \epsilon_{T,t} \sim N(0, \sigma_\epsilon^2)$$

Making use of the definition of total labor force (B.1), rewrite the aggregate production function (B.38) as follows:

$$(B.40) \quad Y_{T,t} = z_{T,t} A_T [(1 - xpat) N_t l_{T,t}^s + xpat N_t l_{T,t}^{ns}]^{\alpha_T} \{ \theta_T [(1 - xpat) N_t K_{T,t-1}]^{-\nu_T} + (1 - \theta_T) (N_t ES_{T,t})^{-\nu_T} \}^{\frac{-(1-\alpha_T)}{\nu_T}}$$

where l_T is the per capita amount of labor, K_T is per capita capital stock and ES_T is energy services in the tradable sector. Collecting N_t :

$$(B.41) \quad Y_{T,t} = z_{T,t} A_T N_t^{\frac{\nu_T(1-\alpha_T)}{\nu_T}} N_t^{\alpha_T} [(1 - xpat) l_{T,t}^s + xpat l_{T,t}^{ns}]^{\alpha_T} \{ \theta_T [(1 - xpat) K_{T,t-1}]^{-\nu_T} + (1 - \theta_T) ES_{T,t}^{-\nu_T} \}^{\frac{-(1-\alpha_T)}{\nu_T}}$$

We can thus express the above production function in per capita terms:

$$(B.42) \quad y_{T,t} = \frac{Y_{T,t}}{N_t} = z_{T,t} A_T [(1 - xpat) l_{T,t}^s + xpat l_{T,t}^{ns}]^{\alpha_T} \{ \theta_T [(1 - xpat) K_{T,t-1}]^{-\nu_T} + (1 - \theta_T) ES_{T,t}^{-\nu_T} \}^{\frac{-(1-\alpha_T)}{\nu_T}}$$

Appendix II: Detailed derivation of equations

Noting that all firms in the model are assumed to be price-takers, the tradable goods firm maximizes the following profits (Π_T):

$$(B.43) \quad \Pi_{T,t} = P_{T,t} y_{T,t} - (1 - xpat) W_{T,t}^s l_{T,t}^s - xpat W_{T,t}^{ns} l_{T,t}^{ns} - (1 - xpat) i_{T,t} K_{T,t-1} - P_{ES,t} ES_{T,t}$$

subject to (B.42). The first order conditions are:

$$(B.44) \quad W_{T,t}^s = \alpha_T P_{T,t} z_{T,t} A_T [(1 - xpat) l_{T,t}^s + xpat l_{T,t}^{ns}]^{\alpha_T - 1} \{ \theta_T [(1 - xpat) K_{T,t-1}]^{-\nu_T} + (1 - \theta_T) ES_{T,t}^{-\nu_T} \}^{\frac{-(1-\alpha_T)}{\nu_T}}$$

$$(B.45) \quad W_{T,t}^{ns} = \alpha_T P_{T,t} z_{T,t} A_T [(1 - xpat) l_{T,t}^s + xpat l_{T,t}^{ns}]^{\alpha_T - 1} \{ \theta_T [(1 - xpat) K_{T,t-1}]^{-\nu_T} + (1 - \theta_T) ES_{T,t}^{-\nu_T} \}^{\frac{-(1-\alpha_T)}{\nu_T}}$$

As the two types of labor inputs are perfectly substitutable, the corresponding marginal productivities are equal, implying:

$$(B.46) \quad W_{T,t}^s = W_{T,t}^{ns}$$

Turning to capital and energy services:

$$(B.47) \quad i_{T,t} = (1 - \alpha_T) \theta_T P_{T,t} z_{T,t} A_T [(1 - xpat) K_{T,t-1}]^{-\nu_T - 1} [(1 - xpat) l_{T,t}^s + xpat l_{T,t}^{ns}]^{\alpha_T} \{ \theta_T [(1 - xpat) K_{T,t-1}]^{-\nu_T} + (1 - \theta_T) ES_{T,t}^{-\nu_T} \}^{\frac{-(1-\alpha_T)}{\nu_T} - 1}$$

$$(B.48) \quad P_{ES,t} = (1 - \alpha_T) (1 - \theta_T) P_{T,t} z_{T,t} A_T ES_{T,t}^{-\nu_T - 1} [(1 - xpat) l_{T,t}^s + xpat l_{T,t}^{ns}]^{\alpha_T} \{ \theta_T [(1 - xpat) K_{T,t-1}]^{-\nu_T} + (1 - \theta_T) ES_{T,t}^{-\nu_T} \}^{\frac{-(1-\alpha_T)}{\nu_T} - 1}$$

As the technology is characterized by constant returns to scale by assumption we have:

$$(B.49) \quad \alpha_T P_{T,t} y_{T,t} = (1 - xpat) W_{T,t}^s l_{T,t}^s + xpat W_{T,t}^{ns} l_{T,t}^{ns}$$

$$(B.50) \quad (1 - \alpha_T) P_{T,t} y_{T,t} = (1 - xpat) i_{T,t} K_{T,t-1} + P_{ES,t} ES_{T,t}$$

so that profits are zero.

B.5 Non-tradable goods firm

The production function of the non-tradable goods firm is

$$(B.51) \quad Y_{NT,t} = z_{NT,t} A_{NT} [L_{NT,t}^s + L_{NT,t}^{ns}]^{\alpha_{NT}} [\theta_{NT} K_{NT,t-1}^{-\nu_{NT}} + (1 - \theta_{NT}) ES_{NT,t}^{-\nu_{NT}}]^{\frac{-(1-\alpha_{NT})}{\nu_{NT}}}$$

where A_{NT} is a technology progress indicator and $z_{NT,t}$ is a shock to productivity which evolves as an AR(1) process:

$$(B.52) \quad \ln z_{NT,t} = \rho_{NT} \ln z_{NT,t-1} + \epsilon_{NT,t} \quad \text{with} \quad \epsilon_{NT,t} \sim N(0, \sigma_{NT}^2)$$

Rewrite (B.52) as follows:

$$(B.53) \quad Y_{NT,t} = z_{NT,t} A_{NT} [(1 - xpat) N_t l_{NT,t}^s + xpat N_t l_{NT,t}^{ns}]^{\alpha_{NT}} \{ \theta_{NT} [(1 - xpat) N_t K_{NT,t-1}]^{-v_{NT}} + (1 - \theta_{NT}) (N_t ES_{NT,t})^{-v_{NT}} \}^{\frac{-(1-\alpha_{NT})}{v_{NT}}}$$

We can thus express the above production function in per capita terms:

$$(B.54) \quad y_{NT,t} = \frac{Y_{NT,t}}{N_t} = z_{NT,t} A_{NT} [(1 - xpat) l_{NT,t}^s + xpat l_{NT,t}^{ns}]^{\alpha_{NT}} \{ \theta_{NT} [(1 - xpat) K_{NT,t-1}]^{-v_{NT}} + (1 - \theta_{NT}) ES_{NT,t}^{-v_{NT}} \}^{\frac{-(1-\alpha_{NT})}{v_{NT}}}$$

The non-tradable goods firm maximizes the following profits:

$$(B.55) \quad \Pi_{NT,t} = P_{NT,t} y_{NT,t} - (1 - xpat) W_{NT,t}^s l_{NT,t}^s - xpat W_{NT,t}^{ns} l_{NT,t}^{ns} - (1 - xpat) i_{NT,t} K_{NT,t-1} - P_{ES,t} ES_{NT,t}$$

subject to (B.54). The first order conditions are:

$$(B.56) \quad W_{NT,t}^s = \alpha_{NT} P_{NT,t} z_{NT,t} A_{NT} [(1 - xpat) l_{NT,t}^s + xpat l_{NT,t}^{ns}]^{\alpha_{NT}-1} \{ \theta_{NT} [(1 - xpat) K_{NT,t-1}]^{-v_{NT}} + (1 - \theta_{NT}) ES_{NT,t}^{-v_{NT}} \}^{\frac{-(1-\alpha_{NT})}{v_{NT}}}$$

$$(B.57) \quad W_{NT,t}^{ns} = \alpha_{NT} P_{NT,t} z_{NT,t} A_{NT} [(1 - xpat) l_{NT,t}^s + xpat l_{NT,t}^{ns}]^{\alpha_{NT}-1} \{ \theta_{NT} [(1 - xpat) K_{NT,t-1}]^{-v_{NT}} + (1 - \theta_{NT}) ES_{NT,t}^{-v_{NT}} \}^{\frac{-(1-\alpha_{NT})}{v_{NT}}}$$

As with the tradable goods sector, the two types of labor inputs are perfectly substitutable and the corresponding marginal productivities are equal, implying:

$$(B.58) \quad W_{NT,t}^s = W_{NT,t}^{ns}$$

Turning to capital and energy services:

$$(B.59) \quad i_{NT,t} = (1 - \alpha_{NT}) \theta_{NT} P_{NT,t} z_{NT,t} A_{NT} [(1 - xpat) K_{NT,t-1}]^{-v_{NT}-1} [(1 - xpat) l_{NT,t}^s + xpat l_{NT,t}^{ns}]^{\alpha_{NT}} \{ \theta_{NT} [(1 - xpat) K_{NT,t-1}]^{-v_{NT}} + (1 - \theta_{NT}) ES_{NT,t}^{-v_{NT}} \}^{\frac{-(1-\alpha_{NT})}{v_{NT}}-1}$$

$$(B.60) \quad P_{ES,t} = (1 - \alpha_{NT}) (1 - \theta_{NT}) P_{NT,t} z_{NT,t} A_{NT} ES_{NT,t}^{-v_{NT}-1} [(1 - xpat) l_{NT,t}^s + xpat l_{NT,t}^{ns}]^{\alpha_{NT}} \{ \theta_{NT} [(1 - xpat) K_{NT,t-1}]^{-v_{NT}} + (1 - \theta_{NT}) ES_{NT,t}^{-v_{NT}} \}^{\frac{-(1-\alpha_{NT})}{v_{NT}}-1}$$

As before, the technology is assumed to be characterized by constant returns to scale, which implies:

$$(B.61) \quad \alpha_{NT} P_{NT,t} y_{NT,t} = (1 - xpat) W_{NT,t}^s l_{NT,t}^s + xpat W_{NT,t}^{ns} l_{NT,t}^{ns}$$

$$(B.62) \quad (1 - \alpha_{NT}) P_{NT,t} y_{NT,t} = (1 - xpat) i_{NT,t} K_{NT,t-1} + P_{ES,t} ES_{NT,t}$$

so that profits are zero.

Appendix II: Detailed derivation of equations

B.6 Energy services firm

The production function of a representative firm producing energy services is assumed as a CES technology with inputs of electricity, oil and gas with an elasticity of input substitution equal to $1 / (1-\lambda)$. Specifically:

$$(B.63) \quad ES_t = [\varphi EL_t^\lambda + \psi O_{ES,t}^\lambda + (1 - \varphi - \psi) G_{ES,t}^\lambda]^{1/\lambda}$$

The firm maximizes the following profits subject to (B.63):

$$(B.64) \quad \Pi_{ES,t} = P_{ES,t} ES_t - P_{EL,t} EL_t - P_{O,t}^d O_{ES,t} - P_{G,t}^d G_{ES,t}$$

The optimal choice of inputs yields the following optimality conditions:

$$(B.65) \quad P_{ES,t} = \left(\frac{EL_t^{1-\lambda}}{\varphi} \right) [\varphi EL_t^\lambda + \psi O_{ES,t}^\lambda + (1 - \varphi - \psi) G_{ES,t}^\lambda]^{\frac{\lambda-1}{\lambda}} P_{EL,t}$$

$$(B.66) \quad P_{ES,t} = \left(\frac{O_{ES,t}^{1-\lambda}}{\psi} \right) [\varphi EL_t^\lambda + \psi O_{ES,t}^\lambda + (1 - \varphi - \psi) G_{ES,t}^\lambda]^{\frac{\lambda-1}{\lambda}} P_{O,t}^d$$

$$(B.67) \quad P_{ES,t} = \left(\frac{G_{ES,t}^{1-\lambda}}{1-\varphi-\psi} \right) [\varphi EL_t^\lambda + \psi O_{ES,t}^\lambda + (1 - \varphi - \psi) G_{ES,t}^\lambda]^{\frac{\lambda-1}{\lambda}} P_{G,t}^d$$

B.7 Electricity generation firm

Electricity is generated using three fuels: oil, gas and renewable energy sources. The production technology is such that a fossil fuels aggregate input and renewables are assumed to be perfectly substitutable inputs. In turn, oil and gas are modeled as inputs to a Cobb-Douglas fossil fuels aggregator, with a unitary elasticity of substitution between oil and gas. Specifically:

$$(B.68) \quad EL_t = O_{EL,t}^\gamma G_{EL,t}^{1-\gamma} + REN_t$$

The firm maximizes the following profits:

$$(B.69) \quad \Pi_{EL,t} = P_{EL,t} EL_t - P_{O,t}^d O_{EL,t} - P_{G,t}^d G_{EL,t} - P_{R,t} REN_t$$

Maximizing (B.69) subject to (B.68) with respect to the optimal choice of inputs yields the following optimality conditions:

$$(B.70) \quad \gamma P_{EL,t} O_{EL,t}^{\gamma-1} G_{EL,t}^{1-\gamma} = P_{O,t}^d$$

$$(B.71) \quad (1 - \gamma) P_{EL,t} O_{EL,t}^\gamma G_{EL,t}^{-\gamma} = P_{G,t}^d$$

The ratio of these two conditions is:

$$(B.72) \quad \frac{\gamma}{1-\gamma} \frac{G_{EL,t}}{O_{EL,t}} = \frac{P_{O,t}^d}{P_{G,t}^d}$$

The optimality conditions for renewables implies that:

$$(B.73) \quad P_{EL,t} = P_{R,t}$$

If we substitute conditions (B.70)-(B.71) and (B.73) into (B.69) we see that the firm earns zero profits.

B.8 Natural resources

Natural resources are given by renewable energy (solar), oil and gas. We assume that they are produced with a simple technology that uses only the input from public capital. Renewable energy is produced according to:

$$(B.74) \quad REN_t = \left[\frac{A_R}{1 + \chi \left(\frac{REN_t}{EL_t} \right)} \right] K_{R,t-1}^g$$

Public capital in renewables follows the recursive equation:

$$(B.75) \quad K_{R,t}^g = I_{R,t}^g + (1 - \delta_R) K_{R,t-1}^g$$

Turning to oil production we have:

$$(B.76) \quad \bar{O}_t = \alpha_O u_t K_{O,t-1}^g$$

Public capital in oil production follows the recursive equation:

$$(B.77) \quad K_{O,t}^g = I_{O,t}^g + (1 - \delta_O) K_{O,t-1}^g$$

We assume gas production is fixed proportional to oil:

$$(B.78) \quad \bar{G}_t = \alpha_G \bar{O}_t$$

Note that, with this formulation, the production of gas also depends on public capital investment through (B.76).

The international price of oil is assumed to evolve over time according to a mean reverting process. The process embeds two adjustment mechanisms: a) a partial adjustment toward the price that would materialize in the long-run if the previous period's level of exports remained forever, and b) a short-run adjustment due to the current change in export levels, as given below:

$$(B.79) \quad \ln P_{O,t}^* = (1 - \varrho_O) \ln P_{O,ss,t-1}^* + \varrho_O \ln P_{O,t-1}^* + \frac{1}{\varepsilon_{SR}} \ln \frac{OX_t}{OX_{t-1}} + \epsilon_{O,t}$$

$$(B.80) \quad \ln \frac{P_{O,ss,t}^*}{P_{O,ss}^*} = \frac{1}{\varepsilon_{LR}} \ln \frac{OX_t}{OX_{ss}}$$

If the short-term price elasticity ε_{SR} and the long-term price elasticity ε_{LR} of residual demand for Saudi oil were both infinite, we return to the standard assumption of a competitive market, and (B.79) simplifies to the usual mean-reverting equation with a constant long-run price, as shown below:

$$(B.81) \quad \ln P_{O,t}^* = (1 - \varrho_O) \ln P_{O,ss}^* + \varrho_O \ln P_{O,t-1}^* + \epsilon_{O,t}$$

The international price of gas is simply assumed to follow an AR(1) process as follows:

$$(B.82) \quad \ln P_{G,t}^* = (1 - \varrho_G) \ln P_{G,ss}^* + \varrho_G \ln P_{G,t-1}^* + \epsilon_{G,t}$$

Appendix II: Detailed derivation of equations

B.9 Government budget constraint

The government budget is:

$$(B.83) \quad \begin{aligned} & \tau_{C,t}P_t[(1-xpat)N_tC_t^s + (xpat)N_tC_t^{ns}] + \tau_{ES,t}P_{ES,t}[(1-xpat)N_tES_t^s + \\ & (xpat)N_tES_t^{ns}] + \tau_{W,t}^s[W_t^s l_t^s + W_{g,t}^s l_{g,t}^s](1-xpat)N_t + \\ & \tau_{W,t}^{ns}W_t^{ns}l_t^{ns}(xpat)N_t + \tau_{REM,t}REM_t(xpat)N_t + \tau_{K,t}[i_{T,t}K_{T,t-1} + \\ & i_{NT,t}K_{NT,t-1}](1-xpat)N_t + P_{O,t}^d(O_{EL,t} + O_{ES,t})N_t + P_{G,t}^d(G_{EL,t} + \\ & G_{ES,t})N_t + P_{R,t}REN_t + x_tP_{O,t}^*(\bar{O}_t - O_{EL,t} - O_{ES,t})N_t + x_tP_{G,t}^*(\bar{G}_t - G_{EL,t} - \\ & G_{ES,t})N_t + [B_t - (1+i_t)B_{t-1}](1-xpat)N_t = P_tI_{O,t}^gN_t + P_tI_{R,t}^gN_t + \\ & W_{g,t}^s l_{g,t}^s(1-xpat)N_t + P_tGC_t(1-xpat)N_t + TR_t(1-xpat)N_t \end{aligned}$$

We can rewrite (B.83) in per capita terms by dividing throughout by N_t so that:

$$(B.84) \quad \begin{aligned} & \tau_{C,t}P_t[(1-xpat)C_t^s + (xpat)C_t^{ns}] + \tau_{ES,t}P_{ES,t}[(1-xpat)ES_t^s + \\ & (xpat)ES_t^{ns}] + \tau_{W,t}^s[W_t^s l_t^s + W_{g,t}^s l_{g,t}^s](1-xpat) + \tau_{W,t}^{ns}W_t^{ns}l_t^{ns}(xpat) + \\ & \tau_{REM,t}REM_t(xpat) + \tau_{K,t}[i_{T,t}K_{T,t-1} + i_{NT,t}K_{NT,t-1}](1-xpat) + P_{O,t}^d(O_{EL,t} + \\ & O_{ES,t}) + P_{G,t}^d(G_{EL,t} + G_{ES,t}) + P_{R,t}REN_t + x_tP_{O,t}^*(\bar{O}_t - O_{EL,t} - O_{ES,t}) + \\ & x_tP_{G,t}^*(\bar{G}_t - G_{EL,t} - G_{ES,t}) + [B_t - (1+i_t)B_{t-1}](1-xpat) = P_tI_{O,t}^g + P_tI_{R,t}^g + \\ & W_{g,t}^s l_{g,t}^s(1-xpat) + P_tGC_t(1-xpat) + TR_t(1-xpat) \end{aligned}$$

B.10 Market equilibrium for energy services

The market clearing condition for energy services is:

$$(B.85) \quad ES_tN_t = ES_t^s(1-xpat)N_t + ES_t^{ns}(xpat)N_t + ES_{T,t}N_t + ES_{NT,t}N_t$$

in per capita terms:

$$(B.86) \quad ES_t = (1-xpat)ES_t^s + (xpat)ES_t^{ns} + ES_{T,t} + ES_{NT,t}$$

B.11 Market equilibrium for non-tradable goods

In equation (A.25) from Appendix A, letting $J = C, I, I_g, GC$ and omitting taxes we get:

$$J_{NT} = (1-\theta_C)\left(\frac{P_{NT}}{P}\right)^{-\sigma_C} J$$

and thus, the market clearing condition for non-tradable goods is

$$(B.87) \quad \begin{aligned} & Y_{NT,t} = (1-\theta_C)\left[\frac{(1+\tau_{NT,t})P_{NT,t}}{\bar{P}_t}\right]^{-\sigma_C} [(1-xpat)N_tC_t^s + (xpat)N_tC_t^{ns}] + \\ & (1-\theta_C)\left(\frac{P_{NT,t}}{P_t}\right)^{-\sigma_C} \left\{ (1-xpat)N_t \left[I_{T,t} + \frac{h_T}{2} \left(\frac{I_{T,t}}{K_{T,t-1}} - \delta_T \right)^2 K_{T,t-1} + I_{NT,t} + \right. \right. \\ & \left. \left. \frac{h_{NT}}{2} \left(\frac{I_{NT,t}}{K_{NT,t-1}} - \delta_{NT} \right)^2 K_{NT,t-1} \right] + N_t(I_{O,t}^g + I_{R,t}^g) + (1-xpat)N_tGC_t \right\} \end{aligned}$$

in per capita terms:

$$(B.88) \quad \begin{aligned} & y_{NT,t} = (1-\theta_C)\left[\frac{(1+\tau_{NT,t})P_{NT,t}}{\bar{P}_t}\right]^{-\sigma_C} [(1-xpat)C_t^s + (xpat)C_t^{ns}] + \\ & (1-\theta_C)\left(\frac{P_{NT,t}}{P_t}\right)^{-\sigma_C} \left\{ (1-xpat) \left[I_{T,t} + \frac{h_T}{2} \left(\frac{I_{T,t}}{K_{T,t-1}} - \delta_T \right)^2 K_{T,t-1} + I_{NT,t} + \right. \right. \\ & \left. \left. \frac{h_{NT}}{2} \left(\frac{I_{NT,t}}{K_{NT,t-1}} - \delta_{NT} \right)^2 K_{NT,t-1} \right] + (I_{O,t}^g + I_{R,t}^g) + (1-xpat)GC_t \right\} \end{aligned}$$

B.12 Aggregate resource constraint

In our model, the budget constraints of Saudi and non-Saudi households and the government, together with the optimality conditions of the households and firms provide the aggregate resource constraint of the economy.

Without loss of generality, in order to limit the clutter of notation, we assume that the tax rates on tradable and non-tradable goods consumption are equal, i.e. $\tau_{T,t} = \tau_{NT,t}$. In addition, we denote the expressions for capital adjustment costs by $AC_{T,t}$ and $AC_{NT,t}$.

We start by adding the budget constraints of Saudi and non-Saudi households multiplied respectively by $(1-xpat)$ and $(xpat)$ to get the terms in aggregate as follows:

$$(B.89) \quad (1-xpat)[x_t B_t^* - (x_{t-1} + x_t i_t^*) B_{t-1}^*] + (1-xpat)[B_t - (1+i_t) B_{t-1}] + (1-xpat)(1+\tau_{C,t})P_t C_t^s + (1-xpat)(1+\tau_{ES,t})P_{ES,t} ES_t^s + (1-xpat)P_t[(I_{T,t} + AC_{T,t}) + (I_{NT,t} + AC_{NT,t})] + (xpat)(1+\tau_{C,t})P_t C_t^{ns} + (xpat)(1+\tau_{ES,t})P_{ES,t} ES_t^{ns} + (xpat)(1+\tau_{REM,t})REM_t = (1-xpat)(1-\tau_{W,t}^s)[W_t^s l_t^s + W_{g,t}^s l_{g,t}^s] + (1-xpat)(1-\tau_{K,t})[i_{T,t} K_{T,t-1} + i_{NT,t} K_{NT,t-1}] + (1-xpat)TR_t + (xpat)(1-\tau_{W,t}^{ns})W_t^{ns} l_t^{ns}$$

Rearranging:

$$(B.90) \quad (1-xpat)[x_t B_t^* - (x_{t-1} + x_t i_t^*) B_{t-1}^*] + (1-xpat)[B_t - (1+i_t) B_{t-1}] + (1+\tau_{C,t})P_t[(1-xpat)C_t^s + (xpat)C_t^{ns}] + (1+\tau_{ES,t})P_{ES,t}[(1-xpat)ES_t^s + (xpat)ES_t^{ns}] + (1-xpat)P_t[(I_{T,t} + AC_{T,t}) + (I_{NT,t} + AC_{NT,t})] + (xpat)(1+\tau_{REM,t})REM_t = (1-xpat)(1-\tau_{W,t}^s)[W_t^s l_t^s + W_{g,t}^s l_{g,t}^s] + (1-xpat)(1-\tau_{K,t})[i_{T,t} K_{T,t-1} + i_{NT,t} K_{NT,t-1}] + (1-xpat)TR_t + (xpat)(1-\tau_{W,t}^{ns})W_t^{ns} l_t^{ns}$$

From the government budget constraint (B.84) we have:

$$(B.91) \quad (1-xpat)TR_t = (1-xpat)[B_t - (1+i_t) B_{t-1}] + \tau_{C,t}P_t[(1-xpat)C_t^s + (xpat)C_t^{ns}] + \tau_{ES,t}P_{ES,t}[(1-xpat)ES_t^s + (xpat)ES_t^{ns}] + (xpat)\tau_{REM,t}REM_t + (1-xpat)\tau_{W,t}^s[W_t^s l_t^s + W_{g,t}^s l_{g,t}^s] + (xpat)\tau_{W,t}^{ns}W_t^{ns} l_t^{ns} + (1-xpat)\tau_{K,t}[i_{T,t} K_{T,t-1} + i_{NT,t} K_{NT,t-1}] + P_{O,t}^d(O_{EL,t} + O_{ES,t}) + P_{G,t}^d(G_{EL,t} + G_{ES,t}) + P_{R,t}REN_t + x_t P_{O,t}^*(\bar{O}_t - O_{EL,t} - O_{ES,t}) + x_t P_{G,t}^*(\bar{G}_t - G_{EL,t} - G_{ES,t}) - P_t I_{O,t}^g - P_t I_{R,t}^g - (1-xpat)P_t GC_t - (1-xpat)W_{g,t}^s l_{g,t}^s$$

Plugging (B.91) into (B.90) yields:

$$(B.92) \quad (1-xpat)[x_t B_t^* - (x_{t-1} + x_t i_t^*) B_{t-1}^*] + (1-xpat)[B_t - (1+i_t) B_{t-1}] + (1+\tau_{C,t})P_t[(1-xpat)C_t^s + (xpat)C_t^{ns}] + (1+\tau_{ES,t})P_{ES,t}[(1-xpat)ES_t^s + (xpat)ES_t^{ns}] + (1-xpat)P_t[(I_{T,t} + AC_{T,t}) + (I_{NT,t} + AC_{NT,t})] + (xpat)(1+\tau_{REM,t})REM_t = (1-xpat)(1-\tau_{W,t}^s)[W_t^s l_t^s + W_{g,t}^s l_{g,t}^s] + (1-xpat)(1-\tau_{K,t})[i_{T,t} K_{T,t-1} + i_{NT,t} K_{NT,t-1}] + (xpat)(1-\tau_{W,t}^{ns})W_t^{ns} l_t^{ns} + (1-xpat)[B_t - (1+i_t) B_{t-1}] + \tau_{C,t}P_t[(1-xpat)C_t^s + (xpat)C_t^{ns}] + \tau_{ES,t}P_{ES,t}[(1-xpat)ES_t^s + (xpat)ES_t^{ns}] + (xpat)\tau_{REM,t}REM_t + (1-xpat)\tau_{W,t}^s[W_t^s l_t^s + W_{g,t}^s l_{g,t}^s] + (xpat)\tau_{W,t}^{ns}W_t^{ns} l_t^{ns} + (1-xpat)\tau_{K,t}[i_{T,t} K_{T,t-1} + i_{NT,t} K_{NT,t-1}] + P_{O,t}^d(O_{EL,t} + O_{ES,t}) + P_{G,t}^d(G_{EL,t} + G_{ES,t}) + P_{R,t}REN_t + x_t P_{O,t}^*(\bar{O}_t - O_{EL,t} - O_{ES,t}) + x_t P_{G,t}^*(\bar{G}_t - G_{EL,t} - G_{ES,t}) - P_t I_{O,t}^g - P_t I_{R,t}^g - (1-xpat)P_t GC_t - (1-xpat)W_{g,t}^s l_{g,t}^s$$

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Several terms cancel out so that, rearranging:

$$(B.93) \quad \begin{aligned} & (1 - xpat)[x_t B_t^* - (x_{t-1} + x_t i_t^*) B_{t-1}^*] + P_t[(1 - xpat)C_t^s + (xpat)C_t^{ns}] + \\ & P_{ES,t}[(1 - xpat)ES_t^s + (xpat)ES_t^{ns}] + (1 - xpat)P_t[(I_{T,t} + AC_{T,t}) + \\ & (I_{NT,t} + AC_{NT,t})] + (xpat)REM_t + P_t I_{O,t}^g + P_t I_{R,t}^g + (1 - xpat)P_t GC_t = \\ & W_{T,t}[(1 - xpat)l_{T,t}^s + (xpat)l_{T,t}^{ns}] + W_{NT,t}[(1 - xpat)l_{NT,t}^s + (xpat)l_{NT,t}^{ns}] + \\ & (1 - xpat)[i_{T,t} K_{T,t-1} + i_{NT,t} K_{NT,t-1}] + P_{O,t}^d(O_{EL,t} + O_{ES,t}) + P_{G,t}^d(G_{EL,t} + \\ & G_{ES,t}) + P_{R,t}REN_t + x_t P_{O,t}^*(\bar{O}_t - O_{EL,t} - O_{ES,t}) + x_t P_{G,t}^*(\bar{G}_t - G_{EL,t} - G_{ES,t}) \end{aligned}$$

Since wages for Saudi and non-Saudi workers are equal (see [B.46] and [B.58]), we derive the tradable and non-tradable goods production as below:

$$\begin{aligned} W_{T,t}[(1 - xpat)l_{T,t}^s + (xpat)l_{T,t}^{ns}] &= \alpha_T P_{T,t} Y_{T,t} \\ (1 - xpat)i_{T,t} K_{T,t-1} &= (1 - \alpha_T) P_{T,t} Y_{T,t} - P_{ES,t} ES_{T,t} \\ W_{NT,t}[(1 - xpat)l_{NT,t}^s + (xpat)l_{NT,t}^{ns}] &= \alpha_{NT} P_{NT,t} Y_{NT,t} \\ (1 - xpat)i_{NT,t} K_{NT,t-1} &= (1 - \alpha_{NT}) P_{NT,t} Y_{NT,t} - P_{ES,t} ES_{NT,t} \end{aligned}$$

Plug these expressions into (B.93):

$$(B.94) \quad \begin{aligned} & (1 - xpat)[x_t B_t^* - (x_{t-1} + x_t i_t^*) B_{t-1}^*] + P_t[(1 - xpat)C_t^s + (xpat)C_t^{ns}] + \\ & P_{ES,t}[(1 - xpat)ES_t^s + (xpat)ES_t^{ns}] + (1 - xpat)P_t[(I_{T,t} + AC_{T,t}) + (I_{NT,t} + \\ & AC_{NT,t})] + (xpat)REM_t + P_t I_{O,t}^g + P_t I_{R,t}^g + (1 - xpat)P_t GC_t = \alpha_T P_{T,t} Y_{T,t} + \\ & (1 - \alpha_T) P_{T,t} Y_{T,t} - P_{ES,t} ES_{T,t} + \alpha_{NT} P_{NT,t} Y_{NT,t} + (1 - \\ & \alpha_{NT}) P_{NT,t} Y_{NT,t} - P_{ES,t} ES_{NT,t} + P_{O,t}^d(O_{EL,t} + O_{ES,t}) + P_{G,t}^d(G_{EL,t} + \\ & G_{ES,t}) + P_{R,t}REN_t + x_t P_{O,t}^*(\bar{O}_t - O_{EL,t} - O_{ES,t}) + x_t P_{G,t}^*(\bar{G}_t - G_{EL,t} - G_{ES,t}) \end{aligned}$$

Using the zero profit conditions of the energy services firm and the electricity generation firm the expressions (B.64) and (B.69) yield:

$$\begin{aligned} P_{ES,t} ES_t &= P_{EL,t} EL_t + P_{O,t}^d O_{ES,t} + P_{G,t}^d G_{ES,t} \\ P_{EL,t} EL_t &= P_{O,t}^d O_{EL,t} + P_{G,t}^d G_{EL,t} + P_{R,t}REN_t \end{aligned}$$

Substituting the latter expression into the former yields:

$$(B.95) \quad P_{ES,t} ES_t = P_{O,t}^d(O_{EL,t} + O_{ES,t}) + P_{G,t}^d(G_{EL,t} + G_{ES,t}) + P_{R,t}REN_t$$

Using (B.85) in (B.95) yields:

$$(B.96) \quad \begin{aligned} P_{ES,t}(ES_{T,t} + ES_{NT,t}) &= P_{O,t}^d(O_{EL,t} + O_{ES,t}) + P_{G,t}^d(G_{EL,t} + G_{ES,t}) + P_{R,t}REN_t - \\ & P_{ES,t}[(1 - xpat)ES_t^s + (xpat)ES_t^{ns}] \end{aligned}$$

Plugging (B.96) into (B.94):

$$(B.97) \quad \begin{aligned} & (1 - xpat)[x_t B_t^* - (x_{t-1} + x_t i_t^*) B_{t-1}^*] + P_t[(1 - xpat)C_t^S + (xpat)C_t^{ns}] + \\ & P_{ES,t}[(1 - xpat)ES_t^S + (xpat)ES_t^{ns}] + (1 - xpat)P_t[(I_{T,t} + AC_{T,t}) + \\ & (I_{NT,t} + AC_{NT,t})] + (xpat)REM_t + P_t I_{O,t}^g + P_t I_{R,t}^g + (1 - xpat)P_t GC_t = \\ & P_{T,t} y_{T,t} + P_{NT,t} y_{NT,t} - P_{O,t}^d(O_{EL,t} + O_{ES,t}) - P_{G,t}^d(G_{EL,t} + G_{ES,t}) - P_{R,t} REN_t + \\ & P_{ES,t}[(1 - xpat)ES_t^S + (xpat)ES_t^{ns}] + P_{O,t}^d(O_{EL,t} + O_{ES,t}) + P_{G,t}^d(G_{EL,t} + \\ & G_{ES,t}) + P_{R,t} REN_t + x_t P_{O,t}^*(\bar{O}_t - O_{EL,t} - O_{ES,t}) + x_t P_{G,t}^*(\bar{G}_t - G_{EL,t} - G_{ES,t}) \end{aligned}$$

Simplifying:

$$(B.98) \quad \begin{aligned} & (1 - xpat)[x_t B_t^* - (x_{t-1} + x_t i_t^*) B_{t-1}^*] + P_t[(1 - xpat)C_t^S + (xpat)C_t^{ns}] + \\ & (1 - xpat)P_t[(I_{T,t} + AC_{T,t}) + (I_{NT,t} + AC_{NT,t})] + (xpat)REM_t + P_t I_{O,t}^g + \\ & P_t I_{R,t}^g + (1 - xpat)P_t GC_t = P_{T,t} y_{T,t} + P_{NT,t} y_{NT,t} + x_t P_{O,t}^*(\bar{O}_t - O_{EL,t} - \\ & O_{ES,t}) + x_t P_{G,t}^*(\bar{G}_t - G_{EL,t} - G_{ES,t}) \end{aligned}$$

Note that the market clearing condition of energy services requires the demand for energy services by households equate to the production of energy services in aggregate terms. Thus, these terms cancel out in the above derivation. However, energy services are final consumption goods. Hence, by adding energy services expenditure to both sides in (B.98), we get the final aggregate demand of the economy:

$$(B.99) \quad \begin{aligned} & P_t[(1 - xpat)C_t^S + (xpat)C_t^{ns}] + P_{ES,t}[(1 - xpat)ES_t^S + (xpat)ES_t^{ns}] + \\ & (1 - xpat)P_t[(I_{T,t} + AC_{T,t}) + (I_{NT,t} + AC_{NT,t})] + P_t(I_{O,t}^g + I_{R,t}^g) + (1 - \\ & xpat)P_t GC_t + (1 - xpat)[x_t B_t^* - (x_{t-1} + x_t i_t^*) B_{t-1}^*] + (xpat)REM_t = \\ & P_{T,t} y_{T,t} + P_{NT,t} y_{NT,t} + P_{ES,t}[(1 - xpat)ES_t^S + (xpat)ES_t^{ns}] + x_t P_{O,t}^*(\bar{O}_t - \\ & O_{EL,t} - O_{ES,t}) + x_t P_{G,t}^*(\bar{G}_t - G_{EL,t} - G_{ES,t}) \end{aligned}$$

The term $\{P_{T,t} y_{T,t} + P_{NT,t} y_{NT,t} + P_{ES,t}[(1 - xpat)ES_t^S + (xpat)ES_t^{ns}] + x_t P_{O,t}^*(\bar{O}_t - O_{EL,t} - O_{ES,t}) + x_t P_{G,t}^*(\bar{G}_t - G_{EL,t} - G_{ES,t})\}$ is the nominal value added.

Thus, the nominal GDP in our model is:

$$(B.100) \quad \begin{aligned} GDP_t &= P_{T,t} y_{T,t} + P_{NT,t} y_{NT,t} + P_{ES,t}[(1 - xpat)ES_t^S + (xpat)ES_t^{ns}] + \\ & x_t P_{O,t}^*(\bar{O}_t - O_{EL,t} - O_{ES,t}) + x_t P_{G,t}^*(\bar{G}_t - G_{EL,t} - G_{ES,t}) \end{aligned}$$

We obtain the trade balance, in nominal terms as:

$$(B.101) \quad TB_t = (1 - xpat)[x_t B_t^* - (x_{t-1} + x_t i_t^*) B_{t-1}^*] + (xpat)REM_t$$

B.13 Foreign interest rate

The foreign interest rate is given by:

$$(B.102) \quad i_t^* = i^{**} + \zeta[\exp(B_{ss}^* - B_{t-1}^*) - 1]$$

B.14 CO₂ emissions

CO₂ emissions are calculated as:

$$(B.103) \quad CO_2 = ef_O(O_{ES} + O_{EL}) + ef_G(G_{ES} + G_{EL})$$

B.15 Normalization

Finally:

$$(B.104) \quad P_{T,t} = x_t P_{T,t}^*$$

where, P_T^* is exogenous and equal to 1.

Appendix III: List of equations

Aggregate tax rate on tradable and non-tradable consumption goods:

$$(1) \quad (1 + \tau_{c,t}) = \left\{ \frac{\theta_c [(1 + \tau_{T,t}) P_{T,t}]^{1-\sigma_c} + (1 - \theta_c) [(1 + \tau_{NT,t}) P_{NT,t}]^{1-\sigma_c}}{\theta_c P_{T,t}^{1-\sigma_c} + (1 - \theta_c) P_{NT,t}^{1-\sigma_c}} \right\}^{\frac{1}{1-\sigma_c}}$$

Aggregate price of tradable and non-tradable consumption goods:

CES aggregator

$$(2a) \quad P_t = \{\theta_c P_{T,t}^{1-\sigma_c} + (1 - \theta_c) P_{NT,t}^{1-\sigma_c}\}^{\frac{1}{1-\sigma_c}}$$

Cobb-Douglas aggregator

$$(2b) \quad P_t = P_{T,t}^{\theta_c} P_{NT,t}^{1-\theta_c}$$

Saudi labor supply in the tradable goods sector:

$$(3) \quad l_{T,t}^s = \theta_L \left(\frac{W_{T,t}}{W_t} \right)^{-\sigma_L} l_t^s$$

Saudi labor supply in the non-tradable goods sector:

$$(4) \quad l_{NT,t}^s = (1 - \theta_L) \left(\frac{W_{NT,t}}{W_t} \right)^{-\sigma_L} l_t^s$$

Non-Saudi labor supply in the tradable goods sector:

$$(5) \quad l_{T,t}^{ns} = \theta_L \left(\frac{W_{T,t}}{W_t} \right)^{-\sigma_L} l_t^{ns}$$

Non-Saudi labor supply in the non-tradable goods sector:

$$(6) \quad l_{NT,t}^{ns} = (1 - \theta_L) \left(\frac{W_{NT,t}}{W_t} \right)^{-\sigma_L} l_t^{ns}$$

Aggregate wage of tradable and non-tradable consumption goods sectors:

$$(7) \quad W_t = \left[\theta_L (W_{T,t})^{1-\sigma_L} + (1 - \theta_L) (W_{NT,t})^{1-\sigma_L} \right]^{\frac{1}{1-\sigma_L}}$$

Saudi household's marginal utility of consumption:

$$(8) \quad MUC_t^s = \frac{\partial U^s}{\partial c_t^s} = \frac{(c_t^s)^{\sigma-1} [(c_t^s)^\sigma + \kappa (ES_t^s)^\sigma]^{\frac{1-\sigma}{\sigma}}}{[(c_t^s)^\sigma + \kappa (ES_t^s)^\sigma]^{\frac{1}{\sigma}} + \alpha_{CG} G C_t}$$

Saudi household's marginal utility of energy services:

$$(9) \quad MUES_t^s = \frac{\partial U^s}{\partial ES_t^s} = \frac{\kappa (ES_t^s)^{\sigma-1} [(c_t^s)^\sigma + \kappa (ES_t^s)^\sigma]^{\frac{1-\sigma}{\sigma}}}{[(c_t^s)^\sigma + \kappa (ES_t^s)^\sigma]^{\frac{1}{\sigma}} + \alpha_{CG} G C_t}$$

Appendix III: List of equations

Saudi household's marginal utility of leisure:

$$(10) \quad MUL_t^s = \frac{\partial U^s}{\partial l_{T,t}^s} = -\eta(l_t^s + l_{g,t}^s)^\nu$$

Dynamic condition for foreign debt:

$$(11) \quad \frac{MUC_t^s}{(1+\tau_{C,t})P_t} x_t = \beta E_t \frac{MUC_{t+1}^s}{(1+\tau_{C,t+1})P_{t+1}} (x_t + x_{t+1} i_{t+1}^*) = 0$$

Dynamic condition for government debt:

$$(12) \quad \frac{MUC_t^s}{(1+\tau_{C,t})P_t} = \beta E_t \frac{MUC_t^s}{(1+\tau_{C,t+1})P_{t+1}} (1 + i_{t+1}) = 0$$

Optimal allocation of consumption and energy services:

$$(13) \quad \frac{MUES_t^s}{MUC_t^s} = \frac{(1+\tau_{ES,t})P_{ES,t}}{(1+\tau_{C,t})P_t}$$

Optimal allocation of consumption and leisure:

$$(14) \quad \frac{-MUL_t^s}{MUC_t^s} = \frac{(1-\tau_{W,t}^s)W_t}{(1+\tau_{C,t})P_t}$$

Fixed capital investment in the tradable goods sector:

$$(15) \quad q_{T,t} = P_t \left[1 + h_T \left(\frac{l_{T,t}}{K_{T,t-1}} - \delta_T \right) \right]$$

Fixed capital investment in the non-tradable goods sector:

$$(16) \quad q_{NT,t} = P_t \left[1 + h_{NT} \left(\frac{l_{NT,t}}{K_{NT,t-1}} - \delta_{NT} \right) \right]$$

Dynamic condition for fixed capital stock in the tradable goods sector:

$$(17) \quad q_{T,t} = E_t \frac{1}{1 + \frac{x_{t+1} i_{t+1}^*}{x_t}} \left[q_{T,t+1} (1 - \delta_T) + (1 - \tau_{K,t+1}) i_{T,t+1} - P_{t+1} \frac{h_T}{2} \left(\frac{l_{T,t+1}}{K_{T,t}} - \delta_T \right) \left(-\frac{l_{T,t+1}}{K_{T,t}} - \delta_T \right) \right]$$

Dynamic condition for fixed capital stock in the non-tradable goods sector:

$$(18) \quad q_{NT,t} = E_t \frac{1}{1 + \frac{x_{t+1} i_{t+1}^*}{x_t}} \left[q_{NT,t+1} (1 - \delta_{NT}) + (1 - \tau_{K,t+1}) i_{NT,t+1} - P_{t+1} \frac{h_{NT}}{2} \left(\frac{l_{NT,t+1}}{K_{NT,t}} - \delta_{NT} \right) \left(-\frac{l_{NT,t+1}}{K_{NT,t}} - \delta_{NT} \right) \right]$$

Saudi household's budget constraint:

$$(19) \quad x_t B_t^* - (x_{t-1} + x_t l_t^*) B_{t-1}^* + B_t - (1 + i_t) B_{t-1} + (1 + \tau_{ES,t}) P_{ES,t} ES_t^s + \\ (1 + \tau_{C,t}) P_t C_t^s + P_t \left[I_{T,t} + \frac{h_T}{2} \left(\frac{I_{T,t}}{K_{T,t-1}} - \delta_T \right)^2 K_{T,t-1} \right] + P_t \left[I_{NT,t} + \frac{h_{NT}}{2} \left(\frac{I_{NT,t}}{K_{NT,t-1}} - \delta_{NT} \right)^2 K_{NT,t-1} \right] = (1 - \tau_{W,t}^s) [W_t^s l_t^s + W_{g,t}^s l_{g,t}^s] + (1 - \tau_{K,t}) [i_{T,t} K_{T,t-1} + i_{NT,t} K_{NT,t-1}] + TR_t$$

Capital accumulation in the tradable goods sector:

$$(20) \quad I_{T,t} = K_{T,t} - (1 - \delta_T) K_{T,t-1}$$

Capital accumulation in the non-tradable goods sector:

$$(21) \quad I_{NT,t} = K_{NT,t} - (1 - \delta_{NT}) K_{NT,t-1}$$

Non-Saudi household's consumption:

$$(22) \quad (1 + \tau_{C,t}) P_t C_t^{ns} = \varphi_C (1 - \tau_{W,t}^{ns}) W_t l_t^{ns}$$

Non-Saudi household's energy services:

$$(23) \quad (1 + \tau_{ES,t}) P_{ES,t} ES_t^{ns} = \varphi_{ES} (1 - \tau_{W,t}^{ns}) W_t l_t^{ns}$$

Non-Saudi household's remittances:

$$(24) \quad (1 + \tau_{REM,t}) REM_t = (1 - \varphi_C - \varphi_{ES}) (1 - \tau_{W,t}^{ns}) W_t l_t^{ns}$$

Production of tradable goods:

$$(25) \quad y_{T,t} = z_{T,t} A_T [(1 - xpat) l_{T,t}^s + xpat l_{T,t}^{ns}]^{\alpha_T} \{ \theta_T [(1 - xpat) K_{T,t-1}]^{-\nu_T} + (1 - \theta_T) ES_{T,t}^{-\nu_T} \}^{\frac{-(1-\alpha_T)}{\nu_T}}$$

Productivity shock in the tradable goods sector:

$$(26) \quad \ln z_{T,t} = \rho_T \ln z_{T,t-1} + \epsilon_{T,t} \quad \text{with} \quad \epsilon_{T,t} \sim N(0, \sigma_T^2)$$

Optimal labor input in tradable goods production:

$$(27) \quad W_{T,t} = \alpha_T P_{T,t} z_{T,t} A_T [(1 - xpat) l_{T,t}^s + xpat l_{T,t}^{ns}]^{\alpha_T - 1} \{ \theta_T [(1 - xpat) K_{T,t-1}]^{-\nu_T} + (1 - \theta_T) ES_{T,t}^{-\nu_T} \}^{\frac{-(1-\alpha_T)}{\nu_T}}$$

Appendix III: List of equations

Optimal capital input in tradable goods production:

$$(28) \quad i_{T,t} = (1 - \alpha_T) \theta_T P_{T,t} Z_{T,t} A_T [(1 - xpat) K_{T,t-1}]^{-v_T-1} [(1 - xpat) l_{T,t}^s + xpat l_{T,t}^{ns}]^{\alpha_T} \{ \theta_T [(1 - xpat) K_{T,t-1}]^{-v_T} + (1 - \theta_T) ES_{T,t}^{-v_T} \}^{\frac{-(1-\alpha_T)-1}{v_T}}$$

Optimal energy services input in tradable goods production:

$$(29) \quad P_{ES,t} = (1 - \alpha_T)(1 - \theta_T) P_{T,t} Z_{T,t} A_T ES_{T,t}^{-v_T-1} [(1 - xpat) l_{T,t}^s + xpat l_{T,t}^{ns}]^{\alpha_T} \{ \theta_T [(1 - xpat) K_{T,t-1}]^{-v_T} + (1 - \theta_T) ES_{T,t}^{-v_T} \}^{\frac{-(1-\alpha_T)-1}{v_T}}$$

Production of non-tradable goods:

$$(30) \quad y_{NT,t} = z_{NT,t} A_{NT} [(1 - xpat) l_{NT,t}^s + xpat l_{NT,t}^{ns}]^{\alpha_{NT}} \{ \theta_{NT} [(1 - xpat) K_{NT,t-1}]^{-v_{NT}} + (1 - \theta_{NT}) ES_{NT,t}^{-v_{NT}} \}^{\frac{-(1-\alpha_{NT})}{v_{NT}}}$$

Productivity shock in the non-tradable goods sector:

$$(31) \quad \ln z_{NT,t} = \varrho_{NT} \ln z_{NT,t-1} + \epsilon_{NT,t} \quad \text{with} \quad \epsilon_{NT,t} \sim N(0, \sigma_{NT}^2)$$

Optimal labor input in non-tradable goods production:

$$(32) \quad W_{NT,t} = \alpha_{NT} P_{NT,t} Z_{NT,t} A_{NT} [(1 - xpat) l_{NT,t}^s + xpat l_{NT,t}^{ns}]^{\alpha_{NT}-1} \{ \theta_{NT} [(1 - xpat) K_{NT,t-1}]^{-v_{NT}} + (1 - \theta_{NT}) ES_{NT,t}^{-v_{NT}} \}^{\frac{-(1-\alpha_{NT})}{v_{NT}}}$$

Optimal capital input in non-tradable goods production:

$$(33) \quad i_{NT,t} = (1 - \alpha_{NT}) \theta_{NT} P_{NT,t} Z_{NT,t} A_{NT} [(1 - xpat) K_{NT,t-1}]^{-v_{NT}-1} [(1 - xpat) l_{NT,t}^s + xpat l_{NT,t}^{ns}]^{\alpha_{NT}} \{ \theta_{NT} [(1 - xpat) K_{NT,t-1}]^{-v_{NT}} + (1 - \theta_{NT}) ES_{NT,t}^{-v_{NT}} \}^{\frac{-(1-\alpha_{NT})}{v_{NT}}-1}$$

Optimal energy services input in non-tradable goods production:

$$(34) \quad P_{ES,t} = (1 - \alpha_{NT})(1 - \theta_{NT}) P_{NT,t} Z_{NT,t} A_{NT} ES_{NT,t}^{-v_{NT}-1} [(1 - xpat) l_{NT,t}^s + xpat l_{NT,t}^{ns}]^{\alpha_{NT}} \{ \theta_{NT} [(1 - xpat) K_{NT,t-1}]^{-v_{NT}} + (1 - \theta_{NT}) ES_{NT,t}^{-v_{NT}} \}^{\frac{-(1-\alpha_{NT})}{v_{NT}}-1}$$

Electricity generation:

$$(35) \quad EL_t = O_{EL,t}^Y G_{EL,t}^{1-\gamma} + REN_t$$

Optimal oil input in electricity generation:

$$(36) \quad \gamma P_{EL,t} O_{EL,t}^{\gamma-1} G_{EL,t}^{1-\gamma} = P_{O,t}^d$$

Optimal allocation of oil and gas in electricity generation:

$$(37) \quad \frac{\gamma}{1-\gamma} \frac{G_{EL,t}}{O_{EL,t}} = \frac{P_{O,t}^d}{P_{G,t}^d}$$

Optimal condition for renewable energy input in electricity generation:

$$(38) \quad P_{EL,t} = P_{R,t}$$

Production of energy services:

$$(39) \quad ES_t = [\varphi EL_t^\lambda + \psi O_{ES,t}^\lambda + (1 - \varphi - \psi) G_{ES,t}^\lambda]^{1/\lambda}$$

Optimal electricity input in energy services production:

$$(40) \quad P_{ES,t} = \left(\frac{EL_t^{1-\lambda}}{\varphi} \right) [\varphi EL_t^\lambda + \psi O_{ES,t}^\lambda + (1 - \varphi - \psi) G_{ES,t}^\lambda]^{\frac{\lambda-1}{\lambda}} P_{EL,t}$$

Optimal oil input in energy services production:

$$(41) \quad P_{ES,t} = \left(\frac{O_{ES,t}^{1-\lambda}}{\psi} \right) [\varphi EL_t^\lambda + \psi O_{ES,t}^\lambda + (1 - \varphi - \psi) G_{ES,t}^\lambda]^{\frac{\lambda-1}{\lambda}} P_{O,t}^d$$

Optimal gas input in energy services production:

$$(42) \quad P_{ES,t} = \left(\frac{G_{ES,t}^{1-\lambda}}{1-\varphi-\psi} \right) [\varphi EL_t^\lambda + \psi O_{ES,t}^\lambda + (1 - \varphi - \psi) G_{ES,t}^\lambda]^{\frac{\lambda-1}{\lambda}} P_{G,t}^d$$

Renewable energy production:

$$(43) \quad REN_t = \left[\frac{A_R}{1 + \chi \left(\frac{REN_t}{EL_t} \right)} \right] K_{R,t-1}^g$$

Public capital accumulation in the renewable energy sector:

$$(44) \quad I_{R,t}^g = K_{R,t}^g - (1 - \delta_R) K_{R,t-1}^g$$

Oil production:

$$(45) \quad \bar{O}_t = \alpha_O u_t K_{O,t-1}^g$$

Public capital accumulation in the oil sector:

$$(46) \quad I_{O,t}^g = K_{O,t}^g - (1 - \delta_O) K_{O,t-1}^g$$

Appendix III: List of equations

Gas production:

$$(47) \quad \bar{G}_t = \alpha_G \bar{O}_t$$

Equilibrium in market for energy services:

$$(48) \quad ES_t = (1 - xpat)ES_t^s + (xpat)ES_t^{ns} + ES_{T,t} + ES_{NT,t}$$

Government budget constraint:

$$(49) \quad \tau_{C,t}P_t[(1 - xpat)C_t^s + (xpat)C_t^{ns}] + \tau_{ES,t}P_{ES,t}[(1 - xpat)ES_t^s + (xpat)ES_t^{ns}] + \tau_{W,t}^s[W_t^s l_t^s + W_{g,t}^s l_{g,t}^s](1 - xpat) + \tau_{W,t}^{ns}W_t^{ns}l_t^{ns}(xpat) + \tau_{REM,t}REM_t(xpat) + \tau_{K,t}[i_{T,t}K_{T,t-1} + i_{NT,t}K_{NT,t-1}](1 - xpat) + P_{O,t}^d(O_{EL,t} + O_{ES,t}) + P_{G,t}^d(G_{EL,t} + G_{ES,t}) + P_{R,t}REN_t + x_t P_{O,t}^*(\bar{O}_t - O_{EL,t} - O_{ES,t}) + x_t P_{G,t}^*(\bar{G}_t - G_{EL,t} - G_{ES,t}) + [B_t - (1 + i_t)B_{t-1}](1 - xpat) = P_t I_{O,t}^g + P_t I_{R,t}^g + W_{g,t}^s l_{g,t}^s(1 - xpat) + P_t GC_t(1 - xpat) + TR_t(1 - xpat)$$

Equilibrium in market for non-tradable goods:

$$(50) \quad y_{NT,t} = (1 - \theta_C) \left[\frac{(1 + \tau_{NT,t})P_{NT,t}}{(1 + \tau_{C,t})P_t} \right]^{-\sigma_C} [(1 - xpat)C_t^s + (xpat)C_t^{ns}] + (1 - \theta_C) \left(\frac{P_{NT,t}}{P_t} \right)^{-\sigma_C} \left\{ (1 - xpat) \left[I_{T,t} + \frac{h_T}{2} \left(\frac{I_{T,t}}{K_{T,t-1}} - \delta_T \right)^2 K_{T,t-1} + I_{NT,t} + \frac{h_{NT}}{2} \left(\frac{I_{NT,t}}{K_{NT,t-1}} - \delta_{NT} \right)^2 K_{NT,t-1} \right] + (I_{O,t}^g + I_{R,t}^g) + (1 - xpat)GC_t \right\}$$

Nominal gross domestic product

$$(51) \quad GDP_t = P_{T,t}Y_{T,t} + P_{NT,t}Y_{NT,t} + P_{ES,t}[(1 - xpat)ES_t^s + (xpat)ES_t^{ns}] + x_t P_{O,t}^*(\bar{O}_t - O_{EL,t} - O_{ES,t}) + x_t P_{G,t}^*(\bar{G}_t - G_{EL,t} - G_{ES,t})$$

Trade balance

$$(52) \quad TB_t = (1 - xpat)[x_t B_t^* - (x_{t-1} + x_t i_t^*)B_{t-1}^*] + (xpat)REM_t$$

Foreign interest rate

$$(53) \quad i_t^* = i^{**} + \zeta[\exp(B_{ss}^* - B_{t-1}^*) - 1]$$

Normalization

$$(54) \quad P_{T,t} = x_t P_{T,t}^*$$

CO₂ emissions

$$(55) \quad CO_2 = ef_O(O_{ES} + O_{EL}) + ef_G(G_{ES} + G_{EL})$$

International price of oil

$$(56) \quad \ln P_{O,t}^* = (1 - \varrho_O) \ln P_{O,ss,t-1}^* + \varrho_O \ln P_{O,t-1}^* + \frac{1}{\varepsilon_{SR}} \ln \frac{OX_t}{OX_{t-1}} + \epsilon_{O,t}$$

$$(57) \quad \ln \frac{P_{O,ss,t}^*}{P_{O,ss}^*} = \frac{1}{\varepsilon_{LR}} \ln \frac{OX_t}{OX_{ss}}$$

International price of gas

$$(58) \quad \ln P_{G,t}^* = (1 - \varrho_G) \ln P_{G,ss}^* + \varrho_G \ln P_{G,t-1}^* + \epsilon_{G,t}$$

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About the Project

The objective of this project is to develop a dynamic stochastic general equilibrium (DSGE) model tailored to the Saudi economy. The model focuses on analyzing the dynamic effects of structural and stochastic shocks, addressing not only the long-term impacts but also the short-term adjustment. Moreover, this project aims to provide a dynamic modeling platform to simulate the impacts of economic reforms within the general framework of Saudi Vision 2030.

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